Interactions and nonlinear relationships

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Agenda

Varying effects by groups

Interactions with continous variables

Nonlinear relationships

1. Varying effects by groups

Heterogeneous treatment effects

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  - Do 2004 voters react differently to social pressure mailer than nonvoters?

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- Two approaches:
  - Difference in effects between groups (subsetting approach)

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  - Do 2004 voters react differently to social pressure mailer than nonvoters?
- Two approaches:
  - Difference in effects between groups (subsetting approach)
  - Interaction terms in regression

```r
social <-read.csv("data/social.csv")

social |>
  filter(messages %in% c("Neighbors", "Control")) |>
  group_by(messages, primary2004) |>
  summarize(avg_vote = mean(primary2006)) |>
  pivot_wider(
    names_from = messages,
    values_from = avg_vote
  ) |>
  mutate(diff_exp_vote_2004 = `Neighbors` - `Control`) |>
  pull(diff_exp_vote_2004)
```

```
## [1] 0.06929617 0.09652525
```

ATE for the nonvoters:

Subset approach

```r
social <-read.csv("data/social.csv")

social |>
  filter(messages %in% c("Neighbors", "Control")) |>
  group_by(messages, primary2004) |>
  summarize(avg_vote = mean(primary2006)) |>
  pivot_wider(
    names_from = messages,
    values_from = avg_vote
  ) |>
  mutate(diff_exp_vote_2004 = `Neighbors` - `Control`) |>
  pull(diff_exp_vote_2004)
```

```
## [1] 0.06929617 0.09652525
```

ATE for the nonvoters: 0.0693

ATE for the voters:

```
social <-read.csv("data/social.csv")

social |>
  filter(messages %in% c("Neighbors", "Control")) |>
  group_by(messages, primary2004) |>
  summarize(avg_vote = mean(primary2006)) |>
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    names_from = messages,
    values_from = avg_vote
  ) |>
  mutate(diff_exp_vote_2004 = `Neighbors` - `Control`) |>
  pull(diff_exp_vote_2004)
```

```
## [1] 0.06929617 0.09652525
```

ATE for the nonvoters: 0.0693

ATE for the voters: 0.0965

How much does the esimated treatment effect differ between groups?

```
diff_exp_vote<- social |>
  filter(messages %in% c("Neighbors", "Control")) |>
  group_by(messages, primary2004) |>
  summarize(avg_vote = mean(primary2006)) |>
  pivot_wider(
    names_from = messages,
    values_from = avg_vote
  ) |>
  mutate(diff_exp_vote_2004 = `Neighbors` - `Control`) |>
  pull(diff_exp_vote_2004)

  diff(diff_exp_vote)[1]
```

```
## [1] 0.02722908
```

- Any easier way to allow for different effects of treatment by groups?

- Can allow for different effects of a variable with an **interaction term**

$$\text{turnout}_i = \alpha + \beta_1 \text{primary2004}_i + \beta_2 \text{neighbors}_i + \beta_3(\text{primary2004}_i \times \text{neighbors}_i) + \epsilon_i$$

- Can allow for different effects of a variable with an **interaction term**

$$\text{turnout}_i = \alpha + \beta_1 \text{primary2004}_i + \beta_2 \text{neighbors}_i + \beta_3(\text{primary2004}_i \times \text{neighbors}_i) + \epsilon_i$$

- Primary 2004 variable multiplied by the neighbors variable

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- Primary 2004 variable multiplied by the neighbors variable
  - Equal to 1 if voted in 2004 (primary2004 $==$ 1) and received neighbors mailer (neighbors $==$ 1)

- Can allow for different effects of a variable with an **interaction term**

$$\text{turnout}_i = \alpha + \beta_1 \text{primary2004}_i + \beta_2 \text{neighbors}_i + \beta_3(\text{primary2004}_i \times \text{neighbors}_i) + \epsilon_i$$

- Primary 2004 variable multiplied by the neighbors variable
    - Equal to 1 if voted in 2004 (primary2004 $==$ 1) and received neighbors mailer (neighbors $==$ 1)
- Easiest to understand by investigating predicted values

Predicted values from non-interacted model

- Let $X_i = \textit{primary}2004_i$ and $Z_i = \textit{neighbors}_i$:

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 X_i + \widehat{\beta}_2 Z_i$$

	Control ($Z_i = 0$)	Neighbors($Z_i = 1$)
no vote ($X_i = 0$)		

Predicted values from non-interacted model

- Let $X_i = \textit{primary}2004_i$ and $Z_i = \textit{neighbors}_i$:

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 X_i + \widehat{\beta}_2 Z_i$$

	Control ($Z_i = 0$)	Neighbors($Z_i = 1$)
no vote ($X_i = 0$)	$\widehat{\alpha}$	

Predicted values from non-interacted model

- Let $X_i = primary2004_i$ and $Z_i = neighbors_i$:

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 X_i + \widehat{\beta}_2 Z_i$$

	Control ($Z_i = 0$)	Neighbors($Z_i = 1$)
no vote ($X_i = 0$)	$\widehat{\alpha}$	$\widehat{\alpha} + \widehat{\beta}_2$
vote ($X_i = 1$)		

- Let $X_i = \textit{primary}2004_i$ and $Z_i = \textit{neighbors}_i$:

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	Control ($Z_i = 0$)	Neighbors($Z_i = 1$)
no vote ($X_i = 0$)	$\widehat{\alpha}$	$\widehat{\alpha} + \widehat{\beta}_2$
vote ($X_i = 1$)	$\widehat{\alpha} + \widehat{\beta}_1$	

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vote ($X_i = 1$)	$\widehat{\alpha} + \widehat{\beta}_1$	$\widehat{\alpha} + \widehat{\beta}_1 + \widehat{\beta}_2$

- Effect of neighbors for non-voters:

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- Effect of neighbors for non-voters: $(\widehat{\alpha} + \widehat{\beta}_2) - (\hat{\alpha}) = \hat{\beta}_2$
- Effect of neighbors for voters:

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no vote ($X_i = 0$)	$\widehat{\alpha}$	$\widehat{\alpha} + \widehat{\beta}_2$
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- Effect of neighbors for voters:$(\widehat{\alpha} + \widehat{\beta}_1 + \widehat{\beta}_2) - (\widehat{\alpha} + \widehat{\beta}_1) = \widehat{\beta}_2$

Now for the interacted model:

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 X_i + \widehat{\beta}_2 Z_i + \widehat{\beta}_3 X_i Z_i$$

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no vote ($X_i = 0$)	$\widehat{\alpha}$	

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- Effect of neighbors for voters: $(\widehat{\alpha} + \widehat{\beta}_1 + \widehat{\beta}_2 + \widehat{\beta}_3) - (\widehat{\alpha} + \widehat{\beta}_1) = \widehat{\beta}_2 + \widehat{\beta}_3$

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 \text{primary2004}_i + \widehat{\beta}_2 \text{neighbors}_i + \widehat{\beta}_3 (\text{primary2004}_i \times \text{neighbors}_i)$$

	Control group	Neighbors group
2004 primary non-voter	$\widehat{\alpha}$	$\widehat{\alpha} + \widehat{\beta}_2$
2004 primary voter	$\widehat{\alpha} + \widehat{\beta}_1$	$\widehat{\alpha} + \widehat{\beta}_1 + \widehat{\beta}_2 + \widehat{\beta}_3$

$\widehat{\alpha}$: turnout rate for 2004 nonvoters in control group

$$\widehat{Y}_i = \widehat{\alpha}+\widehat{\beta}_1\text{primary2004}_i+\widehat{\beta}_2\text{neighbors}_i+\widehat{\beta}_3(\text{primary2004}_i\times\text{neighbors}_i)$$

	Control group	Neighbors group
2004 primary non-voter	$\widehat{\alpha}$	$\widehat{\alpha}+\widehat{\beta}_2$
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$\widehat{\beta}_1$: avg difference in turnout between 2004 voters and nonvoters

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	Control group	Neighbors group
2004 primary non-voter	$\widehat{\alpha}$	$\widehat{\alpha} + \widehat{\beta}_2$
2004 primary voter	$\widehat{\alpha} + \widehat{\beta}_1$	$\widehat{\alpha} + \widehat{\beta}_1 + \widehat{\beta}_2 + \widehat{\beta}_3$

$\widehat{\alpha}$: turnout rate for 2004 nonvoters in control group

$\widehat{\beta}_1$: avg difference in turnout between 2004 voters and nonvoters

$\widehat{\beta}_2$: effect of neighbors for 2004 nonvoters

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 \text{primary2004}_i + \widehat{\beta}_2 \text{neighbors}_i + \widehat{\beta}_3 (\text{primary2004}_i \times \text{neighbors}_i)$$

	Control group	Neighbors group
2004 primary non-voter	$\widehat{\alpha}$	$\widehat{\alpha} + \widehat{\beta}_2$
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$\widehat{\alpha}$: turnout rate for 2004 nonvoters in control group

$\widehat{\beta}_1$: avg difference in turnout between 2004 voters and nonvoters

$\widehat{\beta}_2$: effect of neighbors for 2004 nonvoters

$\widehat{\beta}_3$: difference in the effect of neighbors mailer between 2004 voters & nonvoters

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```
social.neighbor <- social |>
  filter(messages %in% c("Neighbors", "Control"))

fit <- lm(primary2006 ~ primary2004 + messages +
            primary2004:messages, data = social.neighbor)
coef(fit)
```

```
##                  (Intercept)                  primary2004
##                   0.23710990                   0.14869507
##             messagesNeighbors primary2004:messagesNeighbors
##                   0.06929617                   0.02722908
```

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##                   (Intercept)                   primary2004
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##             messagesNeighbors primary2004:messagesNeighbors
##                    0.06929617                    0.02722908
```

- Compare coefficients to subset approach

ATE for nonvoters and voters:

```
## [1] 0.06929617 0.09652525
```

Difference in effects

```
## [1] 0.02722908
```

2. Interactions with Continuous Variables

We will keep using the social pressure experiment with two conditions (only) (neighbors and control)

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Let's first create an age variable

```
social <- social |>
  mutate(age = 2006-yearofbirth)

social.neighbor <- social |>
  filter(messages %in% c("Neighbors", "Control"))
```

- In the previous example:
  - Effect of the neighbors mailer differ for previous vs. nonvoters?

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- How does the effect of the Neighbors mailer varies by age?

Heterogeneous effects

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    - Effect of the neighbors mailer differ for previous vs. nonvoters?
    - Used an interaction term to assess **effect heterogeneity** between groups
- How does the effect of the Neighbors mailer varies by age?
    - Not just two groups, but a continuum of possible age values

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- Remarkably, the same **interaction term** will work here too!

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- How does the effect of the Neighbors mailer varies by age?
  - Not just two groups, but a continuum of possible age values
- Remarkably, the same **interaction term** will work here too!

$$Y_i = \alpha + \beta_1 \text{age}_i + \beta_2 \text{neighbors}_i + \beta_3 (\text{age}_i \times \text{neighbors}_i) + \epsilon_i$$

- Let $X_i = \text{age}_i$ and $Z_i = \text{neighbors}_i$:

Predicted values from non-interacted model

- Let $X_i = \text{age}_i$ and $Z_i = \text{neighbors}_i$:

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 X_i + \widehat{\beta}_2 Z_i$$

	Control group	Neighbors group
25yr old ($X_i = 25$)		

- Let X_i = age$_i$ and Z_i = neighbors$_i$:

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 X_i + \widehat{\beta}_2 Z_i$$

	Control group	Neighbors group
25yr old ($X_i = 25$)	$\widehat{\alpha} + \widehat{\beta}_1 25$	

Predicted values from non-interacted model

- Let X_i = age$_i$ and Z_i = neighbors$_i$:

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 X_i + \widehat{\beta}_2 Z_i$$

	Control group	Neighbors group
25yr old ($X_i = 25$)	$\widehat{\alpha} + \widehat{\beta}_1 25$	$\widehat{\alpha} + \widehat{\beta}_1 25 + \widehat{\beta}_2$
26yr old ($X_i = 26$)		

Predicted values from non-interacted model

- Let X_i = age$_i$ and Z_i = neighbors$_i$:

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 X_i + \widehat{\beta}_2 Z_i$$

	Control group	Neighbors group
25yr old ($X_i = 25$)	$\widehat{\alpha} + \widehat{\beta}_1 25$	$\widehat{\alpha} + \widehat{\beta}_1 25 + \widehat{\beta}_2$
26yr old ($X_i = 26$)	$\widehat{\alpha} + \widehat{\beta}_1 26$	

- Let X_i = age$_i$ and Z_i = neighbors$_i$:

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 X_i + \widehat{\beta}_2 Z_i$$

	Control group	Neighbors group
25yr old ($X_i = 25$)	$\widehat{\alpha} + \widehat{\beta}_1 25$	$\widehat{\alpha} + \widehat{\beta}_1 25 + \widehat{\beta}_2$
26yr old ($X_i = 26$)	$\widehat{\alpha} + \widehat{\beta}_1 26$	$\widehat{\alpha} + \widehat{\beta}_1 26 + \widehat{\beta}_2$

Effect of neighbors for a 25 year-old:

- Let $X_i = \text{age}_i$ and $Z_i = \text{neighbors}_i$:

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 X_i + \widehat{\beta}_2 Z_i$$

	Control group	Neighbors group
25yr old ($X_i = 25$)	$\widehat{\alpha} + \widehat{\beta}_1 25$	$\widehat{\alpha} + \widehat{\beta}_1 25 + \widehat{\beta}_2$
26yr old ($X_i = 26$)	$\widehat{\alpha} + \widehat{\beta}_1 26$	$\widehat{\alpha} + \widehat{\beta}_1 26 + \widehat{\beta}_2$

Effect of neighbors for a 25 year-old:

$$(\widehat{\alpha} + \widehat{\beta}_1 25 + \widehat{\beta}_2) - (\widehat{\alpha} + \widehat{\beta}_1 25) = \widehat{\beta}_2$$

Effect of neighbors for a 26 year-old:

- Let $X_i = \text{age}_i$ and $Z_i = \text{neighbors}_i$:

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 X_i + \widehat{\beta}_2 Z_i$$

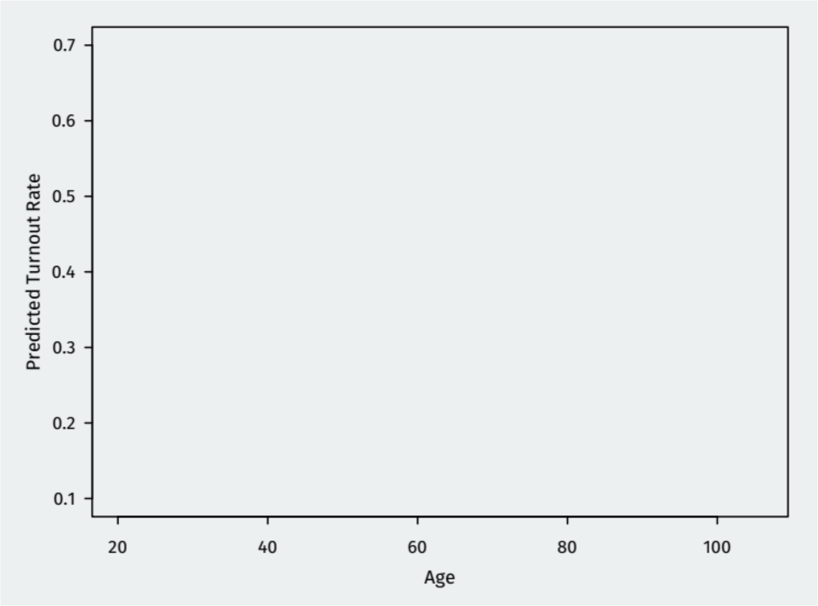
	Control group	Neighbors group
25yr old ($X_i = 25$)	$\widehat{\alpha} + \widehat{\beta}_1 25$	$\widehat{\alpha} + \widehat{\beta}_1 25 + \widehat{\beta}_2$
26yr old ($X_i = 26$)	$\widehat{\alpha} + \widehat{\beta}_1 26$	$\widehat{\alpha} + \widehat{\beta}_1 26 + \widehat{\beta}_2$

Effect of neighbors for a 25 year-old:

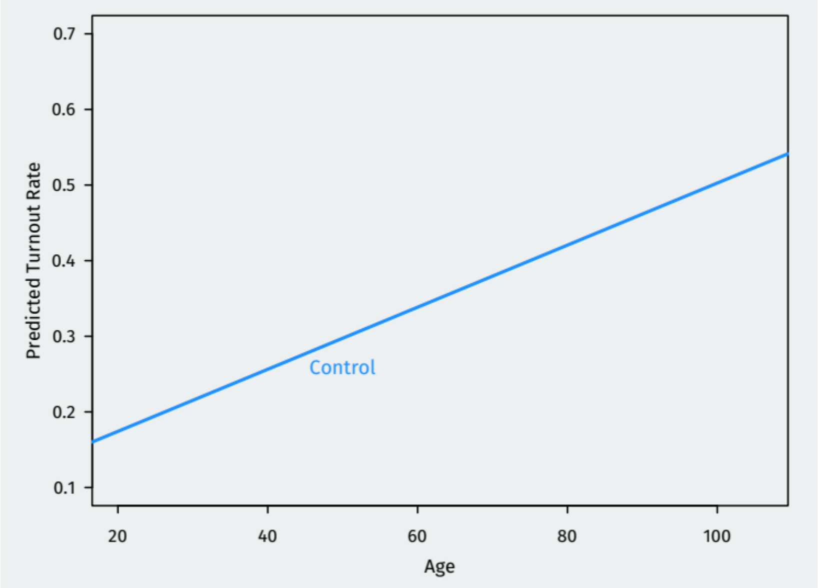
$$(\widehat{\alpha} + \widehat{\beta}_1 25 + \widehat{\beta}_2) - (\widehat{\alpha} + \widehat{\beta}_1 25) = \widehat{\beta}_2$$

Effect of neighbors for a 26 year-old:

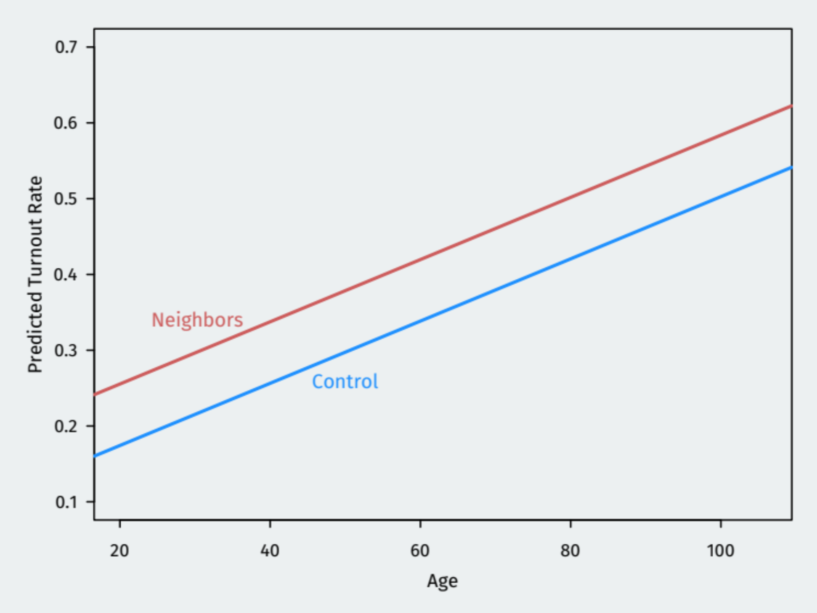
$$(\widehat{\alpha} + \widehat{\beta}_1 26 + \widehat{\beta}_2) - (\widehat{\alpha} + \widehat{\beta}_1 26) = \widehat{\beta}_2$$


0.7
0.6
0.5
0.4
0.3
0.2
0.1
Predicted Turnout Rate
20
40
60
80
100
Age

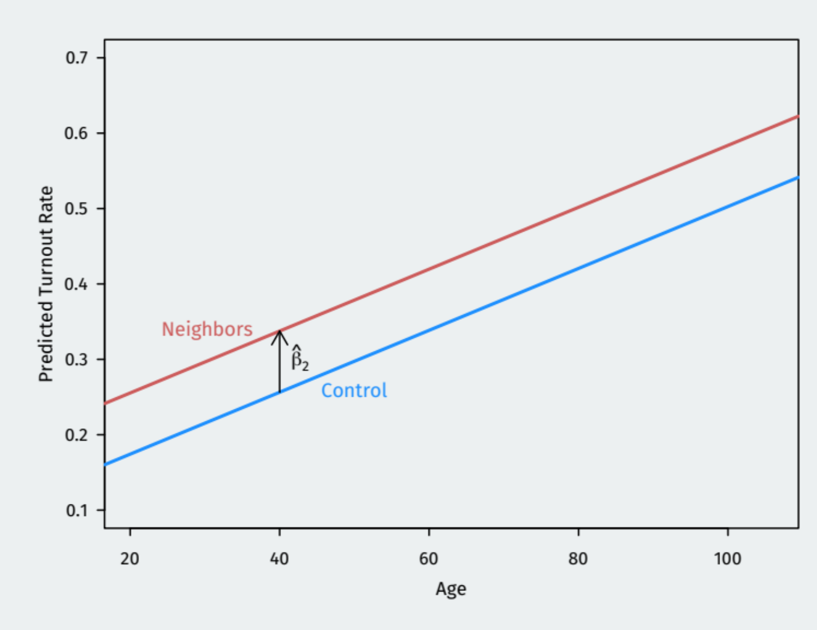
Visualizing the regression



Visualizing the regression



Visualizing the regression



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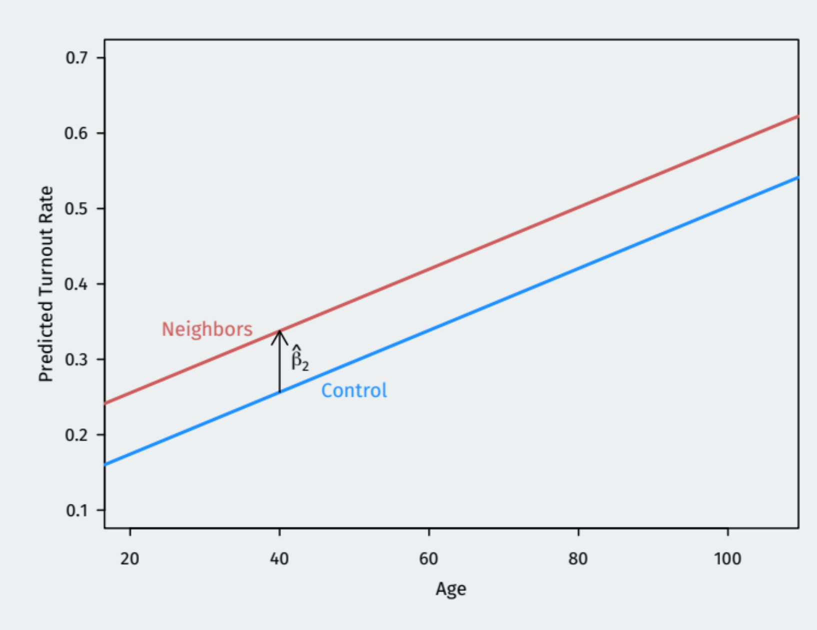

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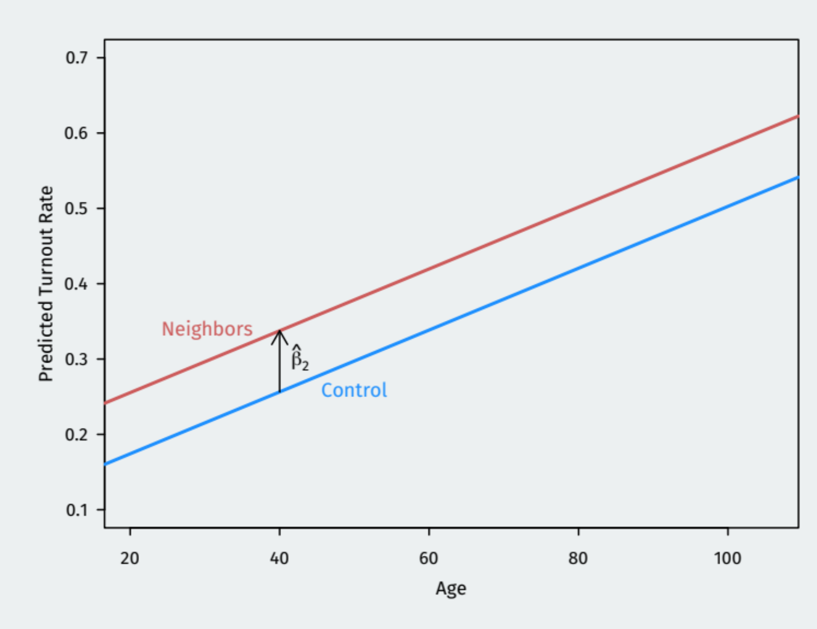

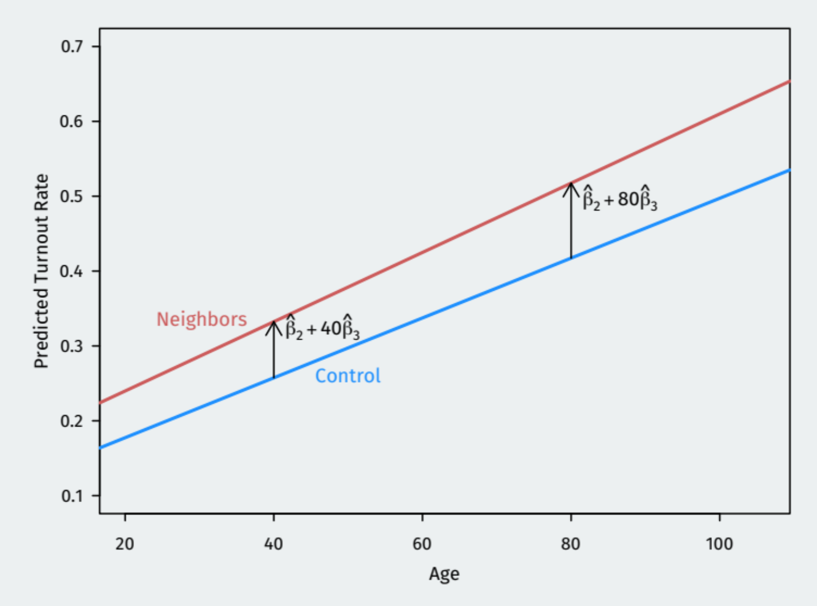
Predicted Turnout Rate
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Neighbors
Control
$\hat{\beta}_2 + 40\hat{\beta}_3$
$\hat{\beta}_2 + 80\hat{\beta}_3$
20
40
60
80
100
Age

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 \text{age}_i + \widehat{\beta}_2 \text{neighbors}_i + \widehat{\beta}_3 (\text{age}_i \times \text{neighbors}_i)$$

- α: average turnout for 0 year-olds in the control group

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 \textsf{age}_i + \widehat{\beta}_2 \textsf{neighbors}_i + \widehat{\beta}_3 (\textsf{age}_i \times \textsf{neighbors}_i)$$

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Interpreting coefficients

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    - Change in effect: $\widehat{\beta}_3$

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```
int.fit <- lm(primary2006 ~ age + messages + age:messages, data = social.neighb
coef(int.fit)
```

```
##           (Intercept)                    age      messagesNeighbors
##          0.0974732574           0.0039982107           0.0498294321
## age:messagesNeighbors
##          0.0006283079
```

- Or you can use the var1 * var2 shortcut, which will add both variable and their interaction:

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##          0.0974732574           0.0039982107           0.0498294321
## age:messagesNeighbors
##          0.0006283079
```

- Or you can use the var1 * var2 shortcut, which will add both variable and their interaction:

```
int.fit2 <- lm(primary2006 ~ age * messages, data = social.neighbor)
coef(int.fit2)
```

```
##           (Intercept)                    age      messagesNeighbors
##          0.0974732574           0.0039982107           0.0498294321
## age:messagesNeighbors
##          0.0006283079
```

General interpretation of interactions

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 X_i + \widehat{\beta}_2 Z_i + \widehat{\beta}_3 X_i Z_i$$

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These hold no matter what types of variables they are!

3. Nonlinear relationship

- We'll look at the Michigan experiment that was trying to see if social pressure affects turnout
- Load the data and create an age variable:

```r
social <- read.csv("data/social.csv")
social$age <- 2006 - social$yearofbirth
summary(social$age)
```

```r
social.neighbor <- social |>
  filter(messages %in% c("Neighbors", "Control"))
```

Linear regression are linear

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Adding a squared term

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- We are now fitting a **parabola** to the data
- In R, we need to wrap the squared term in I():

```
fit.sq <- lm(primary2006 ~ age + I(age^2), social)
coef(fit.sq)
```

```
##   (Intercept)           age      I(age^2)
## -8.168043e-02  1.227357e-02 -8.078954e-05
```

- $\widehat{\beta}_2$: how the effect of age increases as age increases

Predicted values from **lm()**

- We can get predicted values out of R using the predict() function:

```
predict(fit.sq, newdata = list(age = c(20, 21, 22)))
```

```
##         1         2         3
## 0.1314752 0.1404364 0.1492361
```

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- Create a vector of ages to predict and save predictions:

```
age.vals <- 20:85
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Predicted values from lm()

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##         1         2         3
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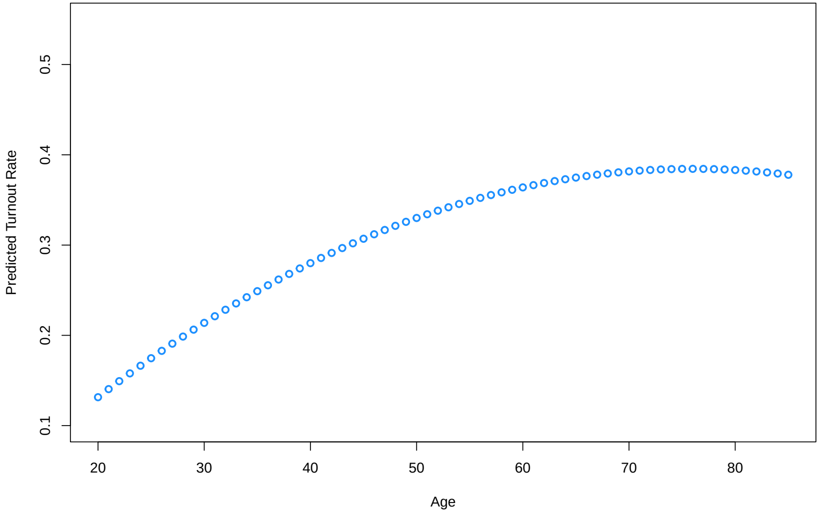
- Create a vector of ages to predict and save predictions:

```
age.vals <- 20:85
age.preds <- predict(fit.sq, newdata = list (age = age.vals
```

- Plot the predictions:

```
plot(x = age.vals, y=age.preds, ylim = c(0.1, 0.55),
     xlab = "Age", ylab = "Predicted Turnout Rate",
     col = "dodgerblue", lwd=2)
```

Plotting predicted values

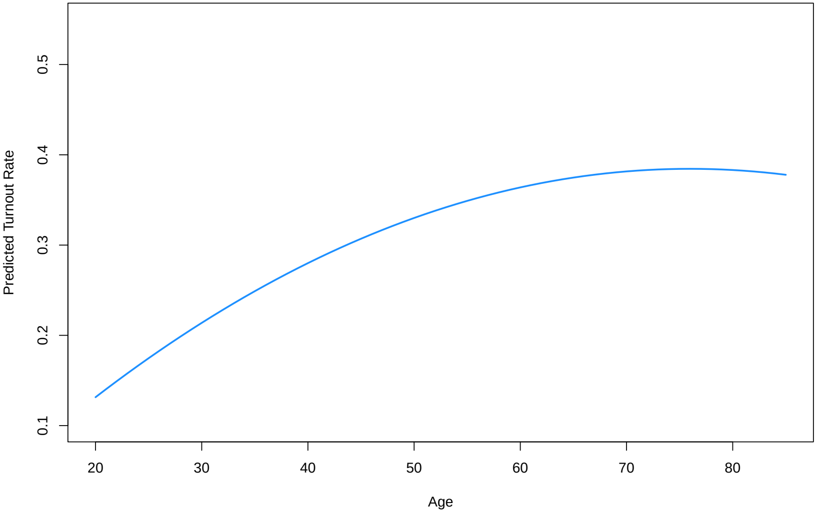


Plotting lines instead of points

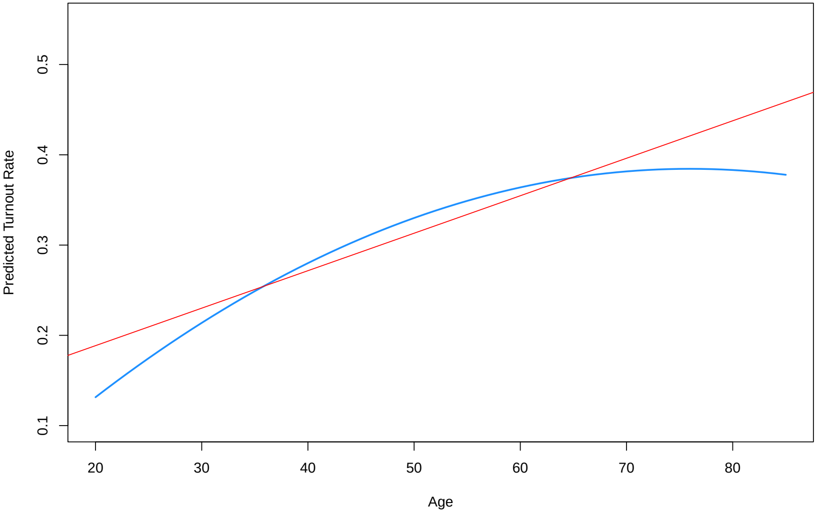
- If you want to connect the dots in your scatterplot, you can use the type = "l" ("line" type)

```
plot(x = age.vals, y=age.preds, ylim = c(0.1, 0.55),
     xlab = "Age", ylab = "Predicted Turnout Rate",
     col = "dodgerblue", lwd=2, type = "l")
```

Plotting predicted values



Comparing to linear fit



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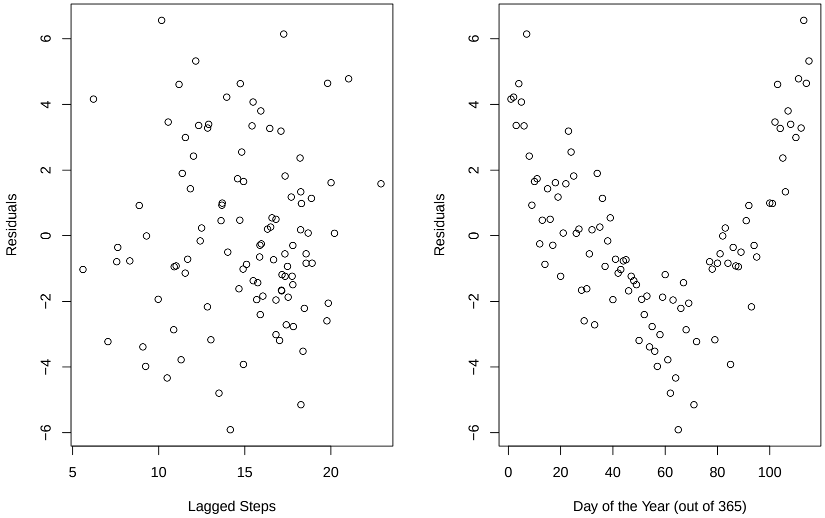
- One independent variable: just look at a scatterplot
- With multiple independent variables, harder to diagnose
- One useful tool: scatterplot of residuals versus independent variables
- Example: health data (weight and step)

```
library(TPDdata)
library(lubridate)

health <- health |>
    mutate(year=year(date)) |>
    mutate(dayofyear = yday(date)) |>
    filter(year==2017) |>
    filter(dayofyear < 120)

health <- drop_na(health)
w.fit <-lm(weight ~ steps_lag  + date, data = health)
```

```
plot(health$steps_lag, residuals(w.fit),
     xlab = "Lagged Steps", ylab = "Residuals")
plot(health$dayofyear, residuals(w.fit),
     xlab = "Day of the Year (out of 365)",
     ylab = "Residuals")
```

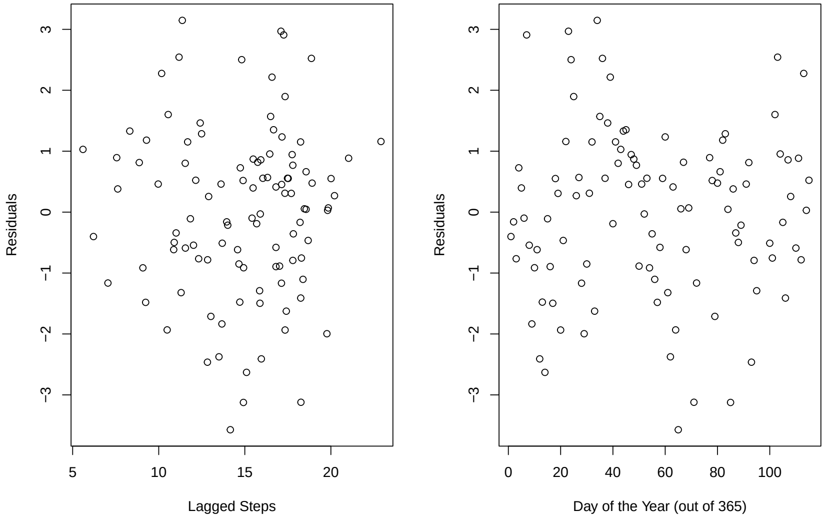

Residuals
6
4
2
0
−2
−4
−6
5
10
15
20
Lagged Steps
Residuals
6
4
2
0
−2
−4
−6
0
20
40
60
80
100
Day of the Year (out of 365)

Add a squared term for a better fit

```r
w.fit.sq <- lm(weight ~ steps_lag+ dayofyear + I(dayofyear^2),
               data=health)
coef(w.fit.sq)
```

```
##    (Intercept)      steps_lag      dayofyear I(dayofyear^2)
##  177.221763772    0.048823891   -0.425866681    0.002210719
```

```r
plot(health$steps_lag, residuals(w.fit.sq),
     xlab = "Lagged Steps", ylab = "Residuals")
plot(health$dayofyear, residuals(w.fit.sq),
     xlab = "Day of the Year (out of 365)",
     ylab = "Residuals")
```


Residuals
3
2
1
0
−1
−2
−3
5
10
15
20
Lagged Steps
Residuals
3
2
1
0
−1
−2
−3
0
20
40
60
80
100
Day of the Year (out of 365)

Class exercises

- We are going to cover some tools for exploring bivariate relationships
- We'll use the data from the Brookckman & Kalla (2016) transphobia study
- Basic summary of experiment:
  - Randomly assigned door-to-door canvassers to two conditions
  - Conditions: perspective-taking script (treatment) or recycling script (placebo)
  - Follow up surveys at 3 days, 3 weeks, 6 weeks, and 3 months

```r
library(tidyverse)
phobia<-read_csv("data/transphobia_all.csv")
phobia <- drop_na(phobia)
phobia <- phobia |>
  mutate(treat_ind = ifelse(treat_ind == "Treat.", 1, 0))
```

Variable	Description
wave	Baseline, 3 days, 21 days, 42 days, and 90 days
age	Age of the respondent in years
female	1=respondent marked “Female” on voter registration, 0 otherwise
voted_gen_14	1 if respondent voted in the 2014 general election
voted_gen_12	1 if respondent voted in the 2012 general election
treat_ind	1 if respondent was assigned to treatment, 0 for control
racename	character name of racial identity indicated on voter file
democrat	1 if respondent is a registered Democrat
therm_trans	0-100 feeling therm. about transgender people
therm_obama	0-100 feeling therm. about Barack Obama

Q 1

Run a regression of thermometer scores for transgender people in wave 1 (3 days) on the treatment indicator (`treat_ind`), the indicator for if the respondent is a Democrat (`democrat`), and the interaction between the two variables

Interpret each of the coefficients in terms of the effects of the intervention

Run a regression of thermometer scores for transgender people in wave 1 (3 days) on the treatment indicator (treat_ind), the indicator fo rif the respondent is a women (female), and the interaction between the two variables.

Interpret each of the coefficients in terms of the effects of the intervention.

Q3

Run a regression of thermometer scores for transgender people in wave 1 on the treatment indicator, age, and the interaction between the two.

What is the estimated effect for a 25 year old? For a 50 year old?

Q4

Run a regression of baseline Obama thermometer scores (therm_obama) on age and the square of age to assess the nonlinear relationship between them.

Calculate predicted values from the model for ages 18 to 90 and plot these as a line