Prediction, iteration, and regression

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Agenda

Prediction

Loops

Evaluating the predictions

Time-series plot

Prediction (again)

Modeling with a line

Linear regression in R

1. Prediction

2016 US Presidential Election

2016 election popular vote

• Clinton: 65,853,516 (48.2%)

• Trump: 62,984,825 (46.1%)

Why did Trump win? **Electoral college**

• Trump: 304, Clinton: 227

Election determined by 77,744 votes (margins in WI, MI, and PA)

0.056% of the electorate (~136 million)

Predicting US Presidential Election

Electoral college system

- Must win an absolute majority of 538 electoral votes
- 538 = 435 (House of Representatives) + 100 (Senators) + 3 (DC)
- Must win at least 270 votes
- Nobody wins an absolute majority → House vote

Must predict winner of each state

Prediction strategy

Predict state-level support for each candidate using polls

Allocate electoral college votes of that state to its predicted winner

Aggregate EC votes across states to determine the predicted winner

Coding strategy:

- For each state, subset to polls within that state
- Further subset the latest polls
- Average the latest polls to estimate support for each candidate
- Allocate the electoral votes to the candidate who has greatest support
- Repeat this for all states and aggregate the electoral votes

Sounds like a lot of subsets :(

2. Loop

A simple example

What if we wanted to know the number of unique values of each column of the cces_2020 data?

```
library(TPDdata)
cces_2020
```

```
## # A tibble: 51,551 x 6
     gender race educ
                                      pid3
                                                 turnout self pres vote
     <fct> <fct> <fct> <fct>
                                      <fct>
                                                        <dbl> <fct>
## 1 Male White 2-year
                                      Republican
                                                            1 Donald J. Trump (~
## 2 Female White Post-grad
                                      Democrat
                                                           NA <NA>
## 3 Female White 4-year
                                      Independent
                                                           1 Joe Biden (Democr~
## 4 Female White 4-year
                                      Democrat
                                                           1 Joe Biden (Democr~
## 5 Male White 4-year
                                      Independent
                                                           1 Other
## 6 Male White Some college
                                                          1 Donald J. Trump (~
                                      Republican
## 7 Male Black Some college
                                      Not sure
                                                           NA <NA>
## 8 Female White Some college
                                      Independent
                                                         1 Donald J. Trump (~
## 9 Female White High school graduate Republican
                                                         1 Donald J. Trump (~
## 10 Female White 4-vear
                                      Democrat
                                                            1 Joe Riden (Democr~
## # ... with 51,541 more rows
```

Manually changing values

```
length(unique(cces_2020$gender))
## [1] 2
length(unique(cces_2020$race))
## [1] 8
length(unique(cces_2020$educ))
## [1] 6
length(unique(cces_2020$pid3))
## [1] 5
length(unique(cces_2020$turnout_self))
## [1] 3
length(unique(cces_2020$pres_vote))
```

[1] 7

Subsetting with brackets

```
Note tat we can also access variables with [[]]:
unique(cces_2020$gender)
## [1] Male Female
## Levels: Male Female skipped not asked
unique(cces_2020[[1]])
## [1] Male
           Female
## Levels: Male Female skipped not asked
unique(cces_2020$pid3)
## [1] Republican Democrat
                              Independent Not sure
                                                     Other
## Levels: Democrat Republican Independent Other Not sure skipped not asked
unique(cces_2020[[4]])
## [1] Republican Democrat Independent Not sure
                                                     Other
## Levels: Democrat Republican Independent Other Not sure skipped not asked
```

Manually changing values, alternative

```
unique(cces_2020[[1]])
## [1] Male Female
## Levels: Male Female skipped not asked
unique(cces_2020[[2]])
## [1] White Black
                              Other Hispanic
## [5] Two or more races Asian Middle Eastern Native American
## 10 Levels: White Black Hispanic Asian Native American ... not asked
unique(cces_2020[[3]])
## [1] 2-year
             Post-grad
                                          4-vear
## [4] Some college High school graduate No HS
## 8 Levels: No HS High school graduate Some college 2-year 4-year ... not aske
unique(cces_2020[[4]])
## [1] Republican Democrat Independent Not sure Other
## Levels: Democrat Republican Independent Other Not sure skipped not asked
unique(cces_2020[[5]])
## [1] 1 NA O
unique(cces_2020[[6]])
```

Recognizing the template

What if you had more values? Not efficient!

Recognize the template:

length(unique(cces_2020[[<<column number>>]]))

Can we give R this template and a set of column numbers have it do our task repeatedly?

for loop provides a way to execute these templates multiple times:

```
output <- rep(NA, tims = ncol(cces_2020)) # 1. output
for (i in seq_along(cces_2020)) { # 2. sequence
  output[i] <- length(unique(cces_2020[[i]])) # 3. body
}
output</pre>
```

[1] 2 8 6 5 3 7

- Elements of a loop:
 - 1 output: vector to hold the
 - 2 i: placeholder name we'll use to swap values between iterations
 - 3 seq_along(cces_2020): vector of values we want the placeholder to take
 - 4 body: a set of expressions that will be repeatedly evaluated
 - 5 {}: curly braces to define beginning and end of the loop
- Indentation is important for readability of the code

2020 polling prediction

Election data: pres20

Variable	Description
state	abbreviated name of state
biden	Biden's vote share (percentage)
trump	Trump's vote share (percentage)
ev	number of electoral college votes for the state

Polling data: polls20

Variable	Description
state end_date daysleft pollster sample_size biden trump	state in which poll was conducted end date the period when poll was conducted number of days between end date and election date name of organization conducting poll number of samples for each poll conducted predicted support for Biden (percentage) predicted support for Trump (percentage)

Some preprocessing

```
library(TPDdata)
# calculate Trump's margin of victory
polls20 <- polls20 |>
 mutate(margin = biden - trump)
pres20 <- pres20 |>
 mutate(margin = biden - trump)
glimpse(polls20)
## Rows: 2,445
## Columns: 8
## $ end date
              <date> 2020-11-02, 2020-11-02, 2020-11-02, 2020-11-02, 2020-11-0~
              <chr> "FL", "PA", "FL", "FL", "NV", "GA", "SC", "MT", "ME", "AZ"~
## $ state
## $ pollster
               <chr> "The Political Matrix/The Listener Group", "Susquehanna", ~
## $ sample_size <dbl> 966, 499, 400, 1054, 1024, 1041, 817, 920, 1024, 610, 1261~
## $ biden
               <dbl> 44.2, 48.4, 47.0, 47.3, 48.4, 45.4, 39.0, 45.0, 52.0, 50.0~
## $ trump
               <dbl> 48.0, 49.2, 48.2, 49.4, 49.1, 49.7, 51.4, 50.0, 40.0, 47.5~
              <dbl> -3.8, -0.8, -1.2, -2.1, -0.7, -4.3, -12.4, -5.0, 12.0, 2.5~
## $ margin
```

Reminder of our goal

- Coding strategy:
 - 1 For each state, subset to polls within that state
 - 2 Further subset the latest polls
 - 3 Average the latest polls to estimate support for each candidate
 - 4 Allocate the electoral votes to the candidate who has greatest support
 - **5** Repeat this for all states and aggregate the electoral votes

Poll prediction for each state

```
poll_pred <- rep(NA, 51) # place holder</pre>
# get list of unique state names to iterate over
state_names <- sort(unique(polls20$state))</pre>
# add labels to holder
names(poll_pred) <- state_names
for (i in 1:51) {
  state_data <- subset(polls20, subset = (state == state_names[i]))</pre>
  latest <- state_data$days_left == min(state_data$days_left)</pre>
  poll_pred[i] <- mean(state_data$margin[latest])</pre>
head(poll_pred)
```

Tidyverse alternative version

```
poll_pred <- polls20 |>
 group_by(state) |>
 filter(days_left == min(days_left)) |>
 summarize(margin_pred = mean(margin))
poll_pred
## # A tibble: 51 x 2
## state margin_pred
##
     <chr>
                <dbl>
## 1 AK
             -9
## 2 AL -26
## 3 AR -23
        4.25
## 4 AZ
## 5 CA
              26
##
   6 CO
              11
## 7 CT
              22
## 8 DC
              89
##
   9 DE
              22
## 10 FL
        0.0800
## # ... with 41 more rows
```

3. Evaluating the predictions

Prediction error = actual outcome - predicted outcome

```
poll_pred <- poll_pred |>
 left_join(pres20) |>
 mutate(errors = margin - margin_pred)
## Joining with `by = join_by(state)`
poll pred
## # A tibble: 51 x 8
     state margin_pred ev biden trump other margin errors
##
##
     <chr>
                <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
   1 AK
                         3 42.8 52.8 0.732 -10.1 -1.06
##
              -9
##
   2 AT.
        -26
                         9 36.6 62.0 0.699 -25.5 0.538
##
   3 AR.
             -23
                         6 34.8 62.4 0.257 -27.6 -4.62
   4 A7.
               4.25
                        11 49.4 49.1 0.263 0.309 -3.94
##
##
   5 CA
              26
                        55 63.5 34.3 0.244 29.2 3.16
##
   6 CO
              11
                         9 55.0 41.6 0.161 13.4 2.41
##
   7 CT
              22
                         7 59.3 39.2 0.129
                                             20.1 -1.93
##
  8 DC
              89
                         3 92.1 5.40 0.491 86.8 -2.25
##
   9 DF.
              22
                         3 58.7 39.8 0.0780 19.0 -3.03
## 10 FI.
              0.0800
                        29 47.9 51.2 0.0835 -3.36 -3.44
## # ... with 41 more rows
```

Assessing the prediction error

Bias: average prediction error

```
mean(poll_pred$errors)
```

[1] -3.983248

Root mean-square error: average magnitude of the prediction error

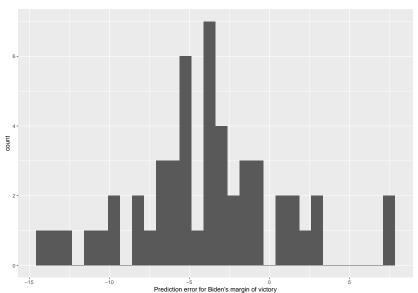
```
sqrt(mean(poll_pred$errors^2))
```

[1] 6.06975

Histogram of the errors

```
ggplot(poll_pred, aes(x = errors)) +
  geom_histogram() +
  labs(
    x = "Prediction error for Biden's margin of victory"
)
```

`stat_bin()` using `bins = 30`. Pick better value with '



Comparing polls to outcome

Sometimes we want plot text labels instead of point and we use geom_text and the label aesthetic:

```
## merge the actual results
ggplot(poll_pred, aes(x = margin_pred, y=margin)) +
  geom_text(aes(label = state)) +
  geom_abline(xintercept = 0, slope = 1, linetype=2) +
  geom_hline(yintercept = 0, color = "grey50") +
  geom_vline(xintercept = 0, color = "grey50")
```

Election prediction: need to predict winner in each state:

[1] 328

Classification problem

- Prediction of binary outcome variable = classification problem
- Wrong prediction \approx misclassification
 - 1 true positive: predict Trump wins when he actually wins
 - 2 false positive: predict Trump wins when he actually loses
 - 3 true negative: predict Trump loses when he actually loses
 - 4 false negative: predict Trump loses when he actually wins
- Sometimes false negatives are more/less important: e.g. civil war

Classification based on polls

Accuracy: sign() returns 1 for a positive number, -1 for a negative number, and 0 for 0

```
poll_pred |>
  summarize(prop_correct = mean(sign(margin_pred) == sign(margin))) |>
  pull()
```

Which states did polls call wrong?

[1] 0.9215686

```
poll_pred |>
           filter(sign(margin_pred) != sign(margin))
## # A tibble: 4 x 8
                             state margin_pred
                                                                                                                                                           ev biden trump other margin errors
##
##
                             <chr>>
                                                                                                   <dbl> <dbl > <db > <
                                                                                                                                                                                                                                                                                                                                              <dbl>
## 1 FL
                                                                                      0.0800
                                                                                                                                                           29 47.9 51.2 0.0835 -3.36 -3.44
## 2 GA
                                                                          -1.15
                                                                                                                                                           16 49.5 49.2 0.0759 0.236 1.39
## 3 NC
                                                                                           3.95
                                                                                                                                                           15 48.6 49.9 0.296 -1.35 -5.30
## 4 NV
                                                                                    -0.350
                                                                                                                                                               6 50.1 47.7 0.759 2.39 2.74
```

4. Time-series plot

National polls

We often want to show a time series of the national-level polls to get a sense of the popular vote:

national_polls20

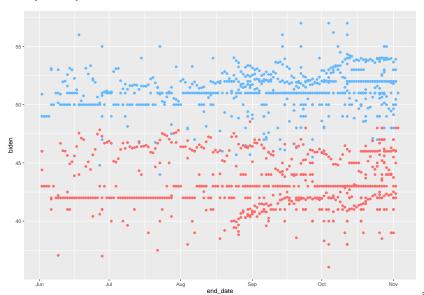
```
## # A tibble: 654 x 5
     end date pollster
                                                            sample_~1 biden trump
##
     <date>
                <chr>
                                                                <dbl> <dbl> <dbl>
   1 2020-11-03 Lake Research
                                                                 2400 51
                                                                            48
  2 2020-11-02 Research Co.
                                                                 1025 50
                                                                            42
## 3 2020-11-02 YouGov
                                                                 1363 53
                                                                            43
                                                                 914 52
                                                                            45
  4 2020-11-02 Ipsos
  5 2020-11-02 SurveyMonkey
                                                                28240 52
                                                                            46
## 6 2020-11-02 HarrisX
                                                                 2297 52
                                                                            48
## 7 2020-11-02 TTPP
                                                                1212 50.4 46.0
## 8 2020-11-02 USC Dornsife
                                                                 5423 53.9 42.4
## 9 2020-11-01 John Zogby Strategies/EMI Research Solutions
                                                                1008 49.6 43.8
## 10 2020-11-01 Swayable
                                                                5174 51.8 46.1
## # ... with 644 more rows, and abbreviated variable name 1: sample_size
```

Plotting the raw results

```
national_polls20 |>
  ggplot(aes(x = end_date)) +
  geom_point(aes(y = biden), color = "steelblue1") +
  geom_point(aes(y = trump), color = "indianred1")
```

Plotting the raw results

Fairly messy:



Clean the mess by taking moving averages

Goal: plot the average of polls in the last 7 days (very difficult with dplyr)

Loop over each day in the data and do:

- 1 subset to all polls in the previous 7 days of that day
- 2 calculate the average of these polls for Biden and Trump
- 3 save the results as a 1-row tibble

You can get R to properly understand dates and do arithmetic with them:

```
head(national_polls20$end_date)
```

```
## [1] "2020-11-03" "2020-11-02" "2020-11-02" "2020-11-02" "2020-11-02" ## [6] "2020-11-02"
```

head(national_polls20\$end_date+3)

```
## [1] "2020-11-06" "2020-11-05" "2020-11-05" "2020-11-05" "2020-11-05" "## [6] "2020-11-05"
```

Lubridate to create dates

We can convert a string to a date using the lubridate package:

```
"2020-11-03" + 3 ## R doesn't know this is a date yet!

## Error in "2020-11-03" + 3: non-numeric argument to binary operator
lubridate::ymd("2020-11-03") + 3

## [1] "2020-11-06"
lubridate::mdy("11/03/2020") + 3
```

[1] "2020-11-06"

Getting a vector of dates

Setup the vector of dates to cover:

```
## [1] "2020-06-03" "2020-06-04" "2020-06-05" "2020-06-06" "2020-06-07" ## [6] "2020-06-08"
```

Moving window loop

```
output <-vector("list", length=length(all_dates))</pre>
for (i in seq_along(all_dates)) {
  this_date <- all_dates[[i]]
  this week <- national polls20 |>
    filter(
      this_date - end_date >= 0, # this_date is after end_date
      this_date - end_date < 7 # within a week
  output[[i]] <- this_week |>
    summarize(
      date = this date,
      biden = mean(biden, na.rm = TRUE),
      trump = mean(trump, na.rm = TRUE)
output <- bind_rows(output)</pre>
```

Result

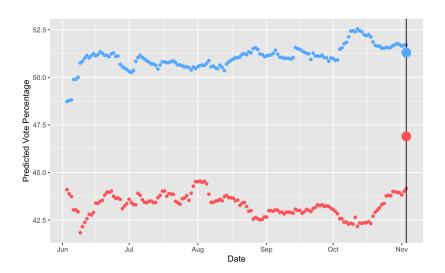
output

[1] 2 8 6 5 3 7

Let's plot

```
output |>
  ggplot(aes(x = date)) +
  geom_point(aes(y = biden), color = "steelblue1") +
  geom_point(aes(y = trump), color = "indianred1") +
  geom_vline(xintercept = election_day) +
  geom_point(aes(x = election_day, y = 51.3), color = "steelblue1", size = 5) +
  geom_point(aes(x = election_day, y = 46.9), color = "indianred1", size = 5) +
  labs(
    x = "Date",
    y = "Predicted Vote Percentage"
)
```

Let's plot



Prediction (again)

Predicting weight

Predicting weight with activity: health data

Variable	Description
date	date of measurements
active_calories	calories burned
steps	number of steps taken (in, 1,000s)
weight	weight (lbs)
steps_lag	steps on day before (in 1,000s)
calories_lag	calories burned on day before

Predicting using bivariate relationship

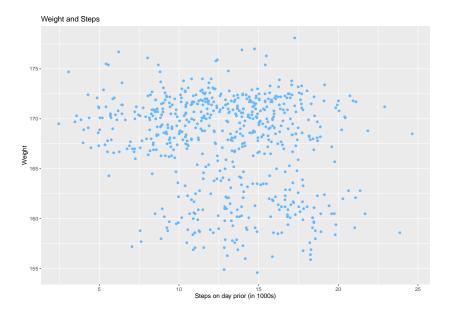
- Goal: what's our best guess about Y_i if we know what X_i is?
 - What's our best guess about one's weight this morning? Would it be helpful if we know how many steps she/he took yesterday?
- Terminology:
 - Dependent/outcome variable: what we want to predict (weight)
 - Independent/explanatory variable: what we are using to predict (steps)

I oad the data:

```
library(TPDdata)
health <- drop_na(health)</pre>
```

Plot the data:

```
ggplot(health, aes(x = steps_lag, y = weight)) +
  geom_point(color = "steelblue1") +
  labs(
    x = "Steps on day prior (in 1000s)"
    y = "Weight",
    title = "Weight and Steps"
)
```



Prediction on variable with another

- Prediction with access to just Y: average of the Y values
- Prediction with another variable: for any value of X, what's the best guess about Y?
 - Need a function y = f(x) that maps values of X into predictions
 - Machine learning: fancy ways to determine f(x)
- Example: what if did 5,000 steps today? What's my best guess about weight?

Start with looking at a narrow strip of X

Let's find all values that round to 5,000 steps:

```
health |>
  filter(round(steps_lag) == 5)

## # A tibble: 12 x 6
```

```
##
                 active calories steps weight steps lag calorie lag
      date
                           <dbl> <dbl>
                                        <dbl>
                                                   <dbl>
##
      <dat.e>
                                                               <dbl>
    1 2015-09-08
                           1111. 15.2
                                         169.
                                                   5.02
                                                                410.
##
   2 2015-12-12
##
                            728. 14.7
                                         167.
                                                   5.36
                                                                259.
##
   3 2015-12-28
                            430. 8.94
                                       170.
                                                   5.19
                                                                314
##
   4 2016-01-29
                            475. 8.26
                                         171.
                                                   4.95
                                                                314.
##
   5 2016-02-14
                            264. 5.42
                                         172.
                                                   4.86
                                                                297.
   6 2016-02-15
                            892. 13.1
                                         171.
                                                   5.42
                                                                264.
##
##
   7 2016-05-02
                            627. 11.8
                                         170.
                                                    5.04
                                                                283.
   8 2016-06-27
                            352. 7.21
                                         169.
                                                   4.93
                                                                212.
##
    9 2016-07-22
                            766. 14.8
                                         167.
                                                   4.96
                                                                251.
##
## 10 2016-11-25
                            452 9.4
                                         173.
                                                   5.26
                                                                295
                                         171.
                                                                304.
## 11 2016-11-28
                            577. 11.8
                                                   4.97
## 12 2016-12-30
                            621, 12,4
                                         176.
                                                    5.42
                                                                371.
```

Best guess about Y for this X

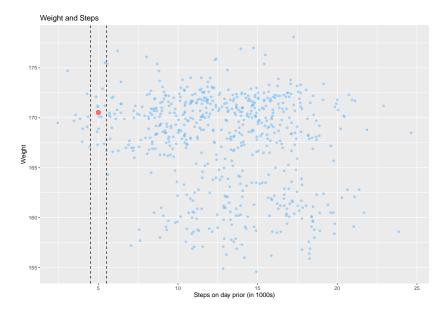
Best prediction about weight for a step count of roughly 5,000 is the average weight for observations around that value:

```
mean_wt_5k_steps <- health |>
  filter(round(steps_lag) == 5) |>
  summarize(mean(weight)) |>
  pull()
mean_wt_5k_steps
```

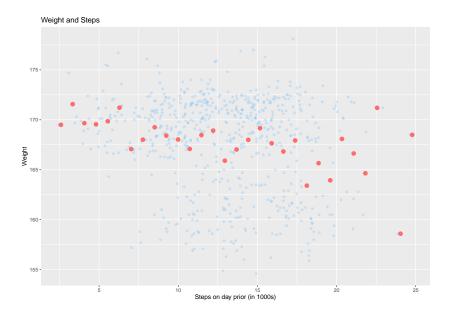
```
## [1] 170.5333
```

Plotting the best guess

```
ggplot(health, aes(x = steps_lag, y = weight)) +
  geom_point(color = "steelblue1", alpha = 0.5) +
  labs(
    x = "Steps on day prior (in 1000s)",
    y = "Weight",
    title = "Weight and Steps") +
  geom_vline(xintercept = c(4.5, 5.5), linetype = "dashed") +
  geom_point(aes(x = 5, y = mean_wt_5k_steps), color = "indianred1",
  size = 3)
```

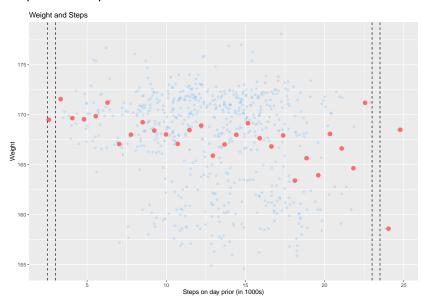


We can use a stat_summary_bin() to add these binned means all over the scatter plot:



But what happens when we make the bins too small?

Gaps and bumps:



Modeling with a line

Using a line to predict

- Can we smooth out these binned means and close gaps? A model
- Simplest possible way to relate two variables: a line

$$y = mx + b$$

- Problem: for any line we draw, not all the data is on the line
 - Some points will be above the line, some below
 - Need a way to account for chance variation away from the line

Linear regression model

• Model for the line of best fit:

$$Y_i = \underbrace{\alpha}_{\text{intercept}} + \underbrace{\beta}_{\text{slope}} \cdot X_i + \underbrace{\epsilon_i}_{\text{error term}}$$

- Coefficients/parameters (α, β) : true unknown intercept/slope of the line of best fit
- Chance error ε_i: accounts for the fact that the line doesn't perfectly fit the data
 - Each observation allowed to be off the regression line
 - Chance errors are 0 on average
- Useful fiction: this model represents the data generating process
 - George Box (British statistician): "all models are wrong, some are useful"

Interpreting the regression line

$$Y_i = \alpha + \beta \cdot X_i + \epsilon_i$$

- Intercept α: average value of Y when X is 0
 - Average weight when I take 0 steps the day prior
- **Slope** β : average change in Y when X increases by one unit
 - Average decrease in weight for each additional 1,000 steps

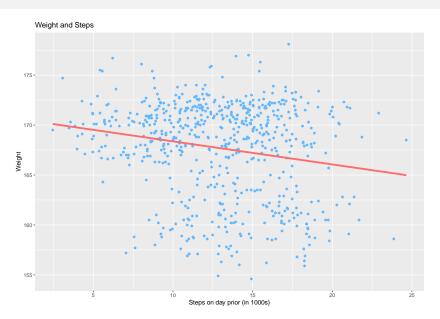
Estimated coefficients

- Parameters: α, β
 - Unknown features of the data-generating process
 - Chance error makes these impossible to observe directly
- Estimates: $\hat{\alpha}, \hat{\beta}$
 - An estimate is our best guess about some parameter
- Regression line: $\hat{Y} = \hat{\alpha} + \hat{\beta} \cdot x$
 - Average value of Y when X is equal to x
 - \bullet Represents the best guess of $\boldsymbol{predicted\ value}$ of the outcome at x

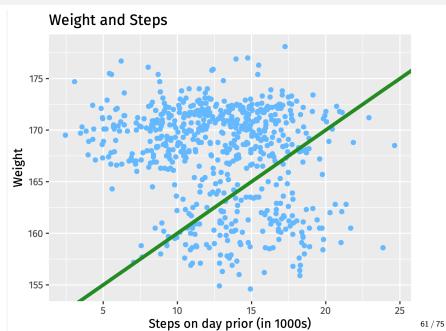
line of best fit

```
ggplot(health, aes(x = steps_lag, y = weight)) +
  geom_point(color = "steelblue1") +
  labs(
    x = "Steps on day prior (in 1000s)",
    y = "Weight",
    title = "Weight and Steps")+
  geom_smooth(method = "lm", se = FALSE, color = "indianred1", size = 1.5)
```

Line of best fit



Why not this line?



Prediction error

Let's understand the **prediction error** for a line with intercept a and slope b

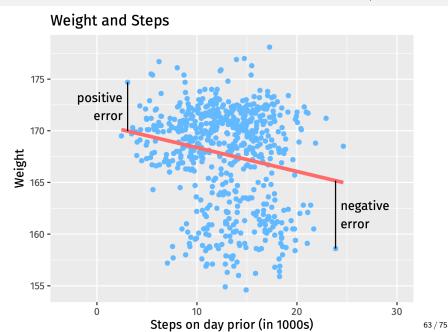
Fitted/predicted value for unit i:

$$a + b \cdot X_i$$

Prediction error (residual):

$$error = actual - predicted = Y_i - (a + b \cdot X_i)$$

Prediction errors/residuals



- Get these estimates by the least squares methods
- Minimize the sum of the squared residuals (SSR):

$$SSR = \sum_{i=1}^{n} (prediction \ error_i)^2 = \sum_{i=1}^{n} (Y_i - a - b \cdot X_i)^2$$

• Finds the line that minimizes the magnitude of the prediction errors!

Linear regression in R

Linear regression in R

- R will calculate least squares line for a data set using Im()
 - Syntax: Im(y x, data = mydata)
 - y is the name of the dependent variable
 - x is the name of the independent variable
 - mydata is the data.frame where they live

```
fit <- lm(weight ~ steps_lag, data=health)
fit

##
## Call:
## lm(formula = weight ~ steps_lag, data = health)
##
## Coefficients:
## (Intercept) steps_lag
## 170.6750 -0.2308</pre>
```

Coefficients

use coef() to extract estimated coefficients:

```
coef(fit)
```

```
## (Intercept) steps_lag
## 170.6749706 -0.2307681
```

Interpretation: a 1-unit increase in X (1,000 steps) is associated with a decrease in the average weight of 0.231 pounds

Question: what would this model predict about the change in average weight for a 10,000 step increase in steps?

The broom package can provide nice summaries of the regression output

augment() can show fitted values, residuals and other unit-level statistics:

```
library(broom)
augment(fit) |>
head()
```

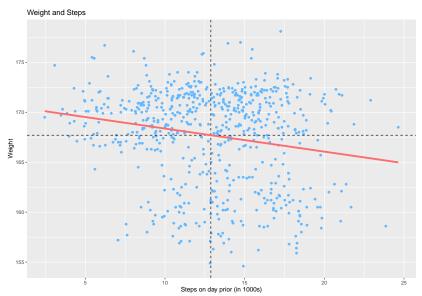
```
## # A tibble: 6 x 8
##
    weight steps_lag .fitted .resid
                                      .hat .sigma
                                                     .cooksd .std.resid
##
     <dbl>
               <dbl>
                      <dbl>
                              <dbl>
                                     <dbl>
                                            <dbl>
                                                       <dbl>
                                                                  <dbl>
## 1
      169.
               17.5
                       167. 2.46
                                   0.00369
                                             4.68 0.000513
                                                                 0.526
      168
               18.4 166. 1.57
                                   0.00463
                                             4.68 0.000264
## 2
                                                                 0.337
## 3
      167.
               19.6 166. 1.05
                                   0.00609
                                             4.68 0.000154
                                                                 0.224
      168.
               10.4
                       168. -0.0750 0.00217
                                             4.68 0.000000280
                                                                -0.0160
## 4
## 5
      168.
               18.7
                       166.
                            1.44
                                   0.00496
                                             4.68 0.000238
                                                                 0.309
## 6
      166.
               9.14
                       169. -2.27
                                   0.00296
                                             4.68 0.000349
                                                                -0.485
```

Properties of least squares

Least squares line always goes through $(\overline{X}, \overline{Y})$

```
ggplot(health, aes(x = steps_lag, y = weight)) +
  geom_point(color = "steelblue1") +
  labs(
    x = "Steps on day prior (in 1000s)",
    y = "Weight",
    title = "Weight and Steps") +
  geom_hline(yintercept = mean(health$weight), linetype = "dashed") +
  geom_vline(xintercept = mean(health$steps_lag), linetype = "dashed") +
  geom_smooth(method = "lm", se = FALSE, color = "indianred1", size = 1.5)
```

Least squares line always goes through $(\overline{X}, \overline{Y})$



Properties of least squares line

Estimated slope is related to correlation:

$$\hat{\beta} =$$
(correlation of X and Y) $\times \frac{\text{SD of Y}}{\text{SD of X}}$

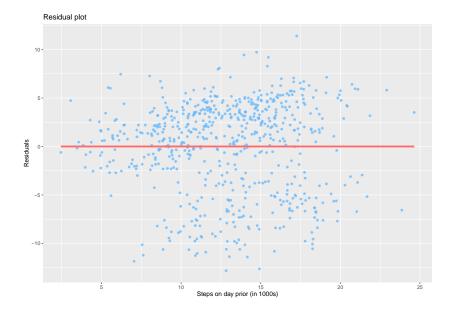
Mean of residuals is always 0

```
augment(fit) |>
  summarize(mean(.resid))
```

```
## # A tibble: 1 x 1
## `mean(.resid)`
## <dbl>
## 1 -1.21e-13
```

Plotting the residuals

```
augment(fit) |>
  ggplot(aes(x = steps_lag, y = .resid)) +
  geom_point(color = "steelblue1", alpha = 0.75) +
  labs(
    x = "Steps on day prior (in 1000s)",
    y = "Residuals",
    title = "Residual plot") +
  geom_smooth(method = "lm", se = FALSE, color = "indianred1", size = 1.5)
```



Smoothed graph of averages

Another way to think of the regression line is a smoothed version of the binned means plot:

