Prediction, iteration, and regression

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1. Prediction

2016 election popular vote

2016 US Presidential Election

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Election determined by 77,744 votes (margins in WI, MI, and PA)

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- 0.056% of the electorate ($\sim$136 million)

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- Must win at least 270 votes
- Nobody wins an absolute majority $\rightsquigarrow$ House vote

Must predict winner of each state

Predict state-level support for each candidate using polls

Prediction strategy

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Allocate electoral college votes of that state to its predicted winner

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- Further subset the latest polls

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Sounds like a lot of subsets :(

2. Loop

What if we wanted to know the number of unique values of each column of the cces_2020 data?

```
library(TPDdata)
cces_2020
```

```
## # A tibble: 51,551 x 6
##    gender race  educ                 pid3        turnout_self pres_vote
##    <fct>  <fct> <fct>                <fct>              <dbl> <fct>
##  1 Male   White 2-year               Republican             1 Donald J. Trump (~
##  2 Female White Post-grad            Democrat              NA <NA>
##  3 Female White 4-year               Independent            1 Joe Biden (Democr~
##  4 Female White 4-year               Democrat               1 Joe Biden (Democr~
##  5 Male   White 4-year               Independent            1 Other
##  6 Male   White Some college         Republican             1 Donald J. Trump (~
##  7 Male   Black Some college         Not sure              NA <NA>
##  8 Female White Some college         Independent            1 Donald J. Trump (~
##  9 Female White High school graduate Republican             1 Donald J. Trump (~
## 10 Female White 4-year               Democrat               1 Joe Biden (Democr~
## # ... with 51,541 more rows
```

```
length(unique(cces_2020$gender))
```

```
## [1] 2
```

```
length(unique(cces_2020$race))
```

```
## [1] 8
```

```
length(unique(cces_2020$educ))
```

```
## [1] 6
```

```
length(unique(cces_2020$pid3))
```

```
## [1] 5
```

```
length(unique(cces_2020$turnout_self))
```

```
## [1] 3
```

```
length(unique(cces_2020$pres_vote))
```

```
## [1] 7
```

Note tat we can also access variables with `[[]]`:

```
unique(cces_2020$gender)
```

```
## [1] Male   Female
## Levels: Male Female skipped not asked
```

```
unique(cces_2020[[1]])
```

```
## [1] Male   Female
## Levels: Male Female skipped not asked
```

```
unique(cces_2020$pid3)
```

```
## [1] Republican  Democrat    Independent Not sure    Other
## Levels: Democrat Republican Independent Other Not sure skipped not asked
```

```
unique(cces_2020[[4]])
```

```
## [1] Republican  Democrat    Independent Not sure    Other
## Levels: Democrat Republican Independent Other Not sure skipped not asked
```

Manually changing values, alternative

```
unique(cces_2020[[1]])
```

```
## [1] Male   Female
## Levels: Male Female skipped not asked
```

```
unique(cces_2020[[2]])
```

```
## [1] White             Black             Other             Hispanic
## [5] Two or more races Asian             Middle Eastern    Native American
## 10 Levels: White Black Hispanic Asian Native American ... not asked
```

```
unique(cces_2020[[3]])
```

```
## [1] 2-year               Post-grad            4-year
## [4] Some college         High school graduate No HS
## 8 Levels: No HS High school graduate Some college 2-year 4-year ... not aske
```

```
unique(cces_2020[[4]])
```

```
## [1] Republican  Democrat    Independent Not sure    Other
## Levels: Democrat Republican Independent Other Not sure skipped not asked
```

```
unique(cces_2020[[5]])
```

```
## [1]  1 NA  0
```

```
unique(cces_2020[[6]])
```

What if you had more values? Not efficient!

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Recognize the template:

```
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Can we give R this template and a set of column numbers have it do our task repeatedly?

for loop provides a way to execute these templates multiple times:

```
output <- rep(NA, tims = ncol(cces_2020))       # 1. output
for (i in seq_along(cces_2020)) {               # 2. sequence
  output[i] <- length(unique(cces_2020[[i]]))  # 3. body
}
output
```

```
## [1] 2 8 6 5 3 7
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- **Indentation** is important for readability of the code

Election data: `pres20`

Variable	Description
state	abbreviated name of state
biden	Biden's vote share (percentage)
trump	Trump's vote share (percentage)
ev	number of electoral college votes for the state

Polling data: `polls20`

Variable	Description
state	state in which poll was conducted
end_date	end date the period when poll was conducted
daysleft	number of days between end date and election date
pollster	name of organization conducting poll
sample_size	number of samples for each poll conducted
biden	predicted support for Biden (percentage)
trump	predicted support for Trump (percentage)

```r
library(TPDdata)

# calculate Trump's margin of victory
polls20 <- polls20 |>
  mutate(margin = biden - trump)

pres20 <- pres20 |>
  mutate(margin = biden - trump)

glimpse(polls20)
```

```
## Rows: 2,445
## Columns: 8
## $ end_date    <date> 2020-11-02, 2020-11-02, 2020-11-02, 2020-11-02, 2020-11-0~
## $ state       <chr> "FL", "PA", "FL", "FL", "NV", "GA", "SC", "MT", "ME", "AZ"~
## $ days_left   <dbl> 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1~
## $ pollster    <chr> "The Political Matrix/The Listener Group", "Susquehanna", ~
## $ sample_size <dbl> 966, 499, 400, 1054, 1024, 1041, 817, 920, 1024, 610, 1261~
## $ biden       <dbl> 44.2, 48.4, 47.0, 47.3, 48.4, 45.4, 39.0, 45.0, 52.0, 50.0~
## $ trump       <dbl> 48.0, 49.2, 48.2, 49.4, 49.1, 49.7, 51.4, 50.0, 40.0, 47.5~
## $ margin      <dbl> -3.8, -0.8, -1.2, -2.1, -0.7, -4.3, -12.4, -5.0, 12.0, 2.5~
```

Reminder of our goal

- Coding strategy:
  1. For each state, subset to polls within that state
  2. Further subset the latest polls
  3. Average the latest polls to estimate support for each candidate
  4. Allocate the electoral votes to the candidate who has greatest support
  5. Repeat this for all states and aggregate the electoral votes

Poll prediction for each state

```
poll_pred <- rep(NA, 51) # place holder

# get list of unique state names to iterate over
state_names <- sort(unique(polls20$state))

# add labels to holder
names(poll_pred) <- state_names

for (i in 1:51) {
  state_data <- subset(polls20, subset = (state == state_names[i]))

  latest <- state_data$days_left == min(state_data$days_left)

  poll_pred[i] <- mean(state_data$margin[latest])
}
head(poll_pred)
```

```
##     AK     AL     AR     AZ     CA     CO
##  -9.00 -26.00 -23.00   4.25  26.00  11.00
```

```
poll_pred <- polls20 |>
  group_by(state) |>
  filter(days_left == min(days_left)) |>
  summarize(margin_pred = mean(margin))
poll_pred
```

```
## # A tibble: 51 x 2
##    state margin_pred
##    <chr>       <dbl>
##  1 AK          -9
##  2 AL         -26
##  3 AR         -23
##  4 AZ           4.25
##  5 CA          26
##  6 CO          11
##  7 CT          22
##  8 DC          89
##  9 DE          22
## 10 FL           0.0800
## # ... with 41 more rows
```

3. Evaluating the predictions

Prediction error = actual outcome - predicted outcome

```
poll_pred <- poll_pred |>
  left_join(pres20) |>
  mutate(errors = margin - margin_pred)
```

```
## Joining with `by = join_by(state)`
```

```
poll_pred
```

```
## # A tibble: 51 x 8
##    state margin_pred    ev biden trump  other  margin errors
##    <chr>       <dbl> <dbl> <dbl> <dbl>  <dbl>   <dbl>  <dbl>
##  1 AK         -9         3  42.8 52.8  0.732  -10.1   -1.06
##  2 AL        -26         9  36.6 62.0  0.699  -25.5    0.538
##  3 AR        -23         6  34.8 62.4  0.257  -27.6   -4.62
##  4 AZ          4.25     11  49.4 49.1  0.263    0.309 -3.94
##  5 CA         26        55  63.5 34.3  0.244   29.2    3.16
##  6 CO         11         9  55.0 41.6  0.161   13.4    2.41
##  7 CT         22         7  59.3 39.2  0.129   20.1   -1.93
##  8 DC         89         3  92.1  5.40 0.491   86.8   -2.25
##  9 DE         22         3  58.7 39.8  0.0780  19.0   -3.03
## 10 FL          0.0800   29  47.9 51.2  0.0835  -3.36  -3.44
## # ... with 41 more rows
```

Bias: average prediction error

```
mean(poll_pred$errors)
```

```
## [1] -3.983248
```

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```
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```

```
## [1] -3.983248
```

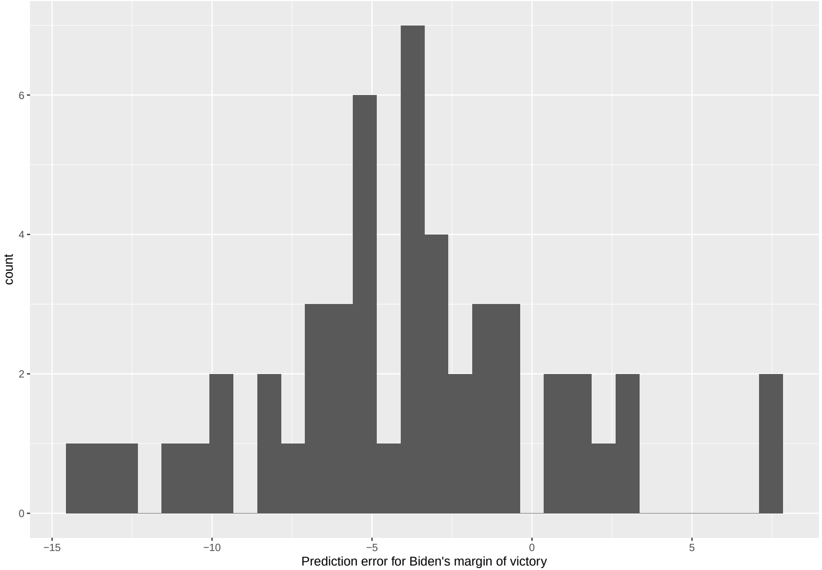
Root mean-square error: average magnitude of the prediction error

```
sqrt(mean(poll_pred$errors^2))
```

```
## [1] 6.06975
```

```r
ggplot(poll_pred, aes(x = errors)) +
  geom_histogram() +
  labs(
    x = "Prediction error for Biden's margin of victory"
  )
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `
```



Sometimes we want plot text labels instead of point and we use `geom_text` and the label aesthetic:

```
## merge the actual results
ggplot(poll_pred, aes(x = margin_pred, y=margin)) +
  geom_text(aes(label = state)) +
  geom_abline(xintercept = 0, slope = 1, linetype=2) +
  geom_hline(yintercept = 0, color = "grey50") +
  geom_vline(xintercept = 0, color = "grey50")
```

Election prediction: need to predict winner in each state:

```
poll_pred |>
  filter(margin > 0) |>
          summarize(sum(ev)) |>
  pull()
```

```
## [1] 306
```

```
poll_pred |>
  filter(margin_pred > 0) |>
  summarize(sum(ev)) |>
  pull()
```

```
## [1] 328
```

Classification problem

- Prediction of binary outcome variable = **classification problem**
- Wrong prediction $\approx$ misclassification
  1. **true positive**: predict Trump wins when he actually wins

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  1. **true positive**: predict Trump wins when he actually wins
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  4. **false negative**: predict Trump loses when he actually wins

- Prediction of binary outcome variable = **classification problem**
- Wrong prediction ≈ misclassification
  1. **true positive**: predict Trump wins when he actually wins
  2. **false positive**: predict Trump wins when he actually loses
  3. **true negative**: predict Trump loses when he actually loses
  4. **false negative**: predict Trump loses when he actually wins
- Sometimes false negatives are more/less important: e.g. civil war

Accuracy: `sign()` returns 1 for a positive number, -1 for a negative number, and 0 for 0

```
poll_pred |>
  summarize(prop_correct = mean(sign(margin_pred) == sign(margin))) |>
  pull()
```

```
## [1] 0.9215686
```

Accuracy: `sign()` returns 1 for a positive number, -1 for a negative number, and 0 for 0

```
poll_pred |>
  summarize(prop_correct = mean(sign(margin_pred) == sign(margin))) |>
  pull()
```

```
## [1] 0.9215686
```

Which states did polls call wrong?

```
poll_pred |>
  filter(sign(margin_pred) != sign(margin))
```

```
## # A tibble: 4 x 8
##   state margin_pred    ev biden trump  other margin errors
##   <chr>       <dbl> <dbl> <dbl> <dbl>  <dbl>  <dbl>  <dbl>
## 1 FL         0.0800    29  47.9  51.2 0.0835 -3.36   -3.44
## 2 GA        -1.15      16  49.5  49.2 0.0759  0.236   1.39
## 3 NC         3.95      15  48.6  49.9 0.296  -1.35   -5.30
## 4 NV        -0.350      6  50.1  47.7 0.759   2.39    2.74
```

4. Time-series plot

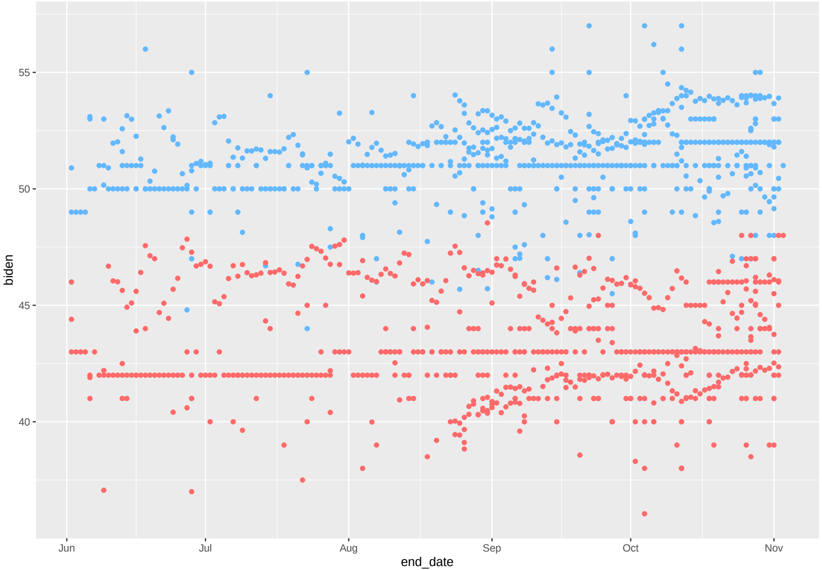
We often want to show a time series of the national-level polls to get a sense of the popular vote:

```
national_polls20
```

```
## # A tibble: 654 x 5
##    end_date   pollster                                    sample_~1 biden trump
##    <date>     <chr>                                           <dbl> <dbl> <dbl>
##  1 2020-11-03 Lake Research                                    2400  51    48
##  2 2020-11-02 Research Co.                                     1025  50    42
##  3 2020-11-02 YouGov                                           1363  53    43
##  4 2020-11-02 Ipsos                                             914  52    45
##  5 2020-11-02 SurveyMonkey                                    28240  52    46
##  6 2020-11-02 HarrisX                                          2297  52    48
##  7 2020-11-02 TIPP                                             1212  50.4  46.0
##  8 2020-11-02 USC Dornsife                                     5423  53.9  42.4
##  9 2020-11-01 John Zogby Strategies/EMI Research Solutions     1008  49.6  43.8
## 10 2020-11-01 Swayable                                         5174  51.8  46.1
## # ... with 644 more rows, and abbreviated variable name 1: sample_size
```

```
national_polls20 |>
  ggplot(aes(x = end_date)) +
  geom_point(aes(y = biden), color = "steelblue1") +
  geom_point(aes(y = trump), color = "indianred1")
```

Fairly messy:



Clean the mess by taking moving averages

Goal: plot the average of polls in the last 7 days (very difficult with `dplyr`)

Loop over each day in the data and do:

1. subset to all polls in the previous 7 days of that day

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Clean the mess by taking moving averages

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Loop over each day in the data and do:

1. subset to all polls in the previous 7 days of that day
2. calculate the average of these polls for Biden and Trump
3. save the results as a 1-row tibble

You can get R to properly understand dates and do arithmetic with them:

```
head(national_polls20$end_date)
```

```
## [1] "2020-11-03" "2020-11-02" "2020-11-02" "2020-11-02" "2020-11-02"
## [6] "2020-11-02"
```

```
head(national_polls20$end_date+3)
```

```
## [1] "2020-11-06" "2020-11-05" "2020-11-05" "2020-11-05" "2020-11-05"
## [6] "2020-11-05"
```

We can convert a string to a date using the `lubridate` package:

```
"2020-11-03" + 3 ## R doesn't know this is a date yet!
```

```
## Error in "2020-11-03" + 3: non-numeric argument to binary operator
```

```
lubridate::ymd("2020-11-03") + 3
```

```
## [1] "2020-11-06"
```

```
lubridate::mdy("11/03/2020") + 3
```

```
## [1] "2020-11-06"
```

Setup the vector of dates to cover:

```
election_day <- lubridate::ymd("2020-11-03")
all_dates <- seq(from = min(national_polls20$end_date) +1,
                 to = election_day,
                 by = "days")
head(all_dates)
```

```
## [1] "2020-06-03" "2020-06-04" "2020-06-05" "2020-06-06" "2020-06-07"
## [6] "2020-06-08"
```

```r
output <-vector("list", length=length(all_dates))
for (i in seq_along(all_dates)) {
  this_date <- all_dates[[i]]

  this_week <- national_polls20 |>
    filter(
      this_date - end_date >= 0,    # this_date is after end_date
      this_date - end_date < 7      # within a week
    )

  output[[i]] <- this_week |>
    summarize(
      date = this_date,
      biden = mean(biden, na.rm = TRUE),
      trump = mean(trump, na.rm = TRUE)
    )
}
output <- bind_rows(output)
```

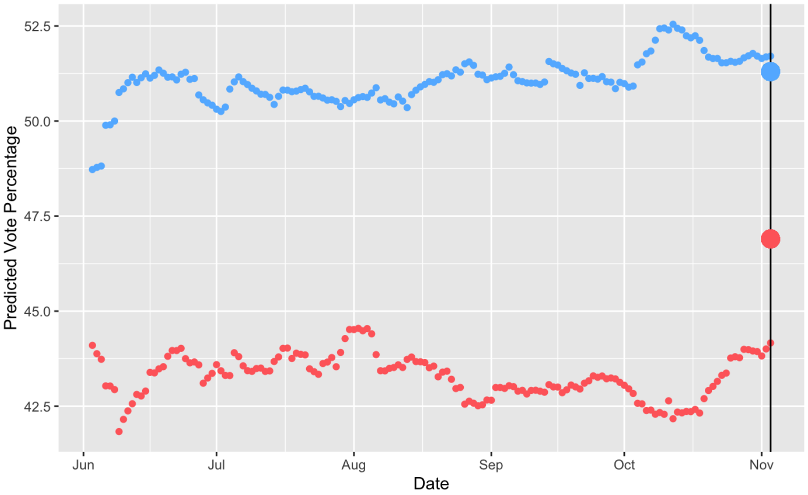
Result

```
output
```

```
## [1] 2 8 6 5 3 7
```

```r
output |>
  ggplot(aes(x = date)) +
  geom_point(aes(y = biden), color = "steelblue1") +
  geom_point(aes(y = trump), color = "indianred1") +
  geom_vline(xintercept = election_day) +
  geom_point(aes(x = election_day, y = 51.3), color = "steelblue1", size = 5) +
  geom_point(aes(x = election_day, y = 46.9), color = "indianred1", size = 5) +
  labs(
    x = "Date",
    y = "Predicted Vote Percentage"
  )
```

Let's plot



52.5
50.0
47.5
45.0
42.5
Predicted Vote Percentage
Jun
Jul
Aug
Sep
Oct
Nov
Date

Prediction (again)

Predicting weight with activity: `health` data

Variable	Description
date	date of measurements
active_calories	calories burned
steps	number of steps taken (in, 1,000s)
weight	weight (lbs)
steps_lag	steps on day before (in 1,000s)
calories_lag	calories burned on day before

- Goal: what's our best guess about Y_i if we know what X_i is?

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    - What's our best guess about one's weight this morning? Would it be helpful if we know how many steps she/he took yesterday?

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Predicting using bivariate relationship

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  - What's our best guess about one's weight this morning? Would it be helpful if we know how many steps she/he took yesterday?
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  - **Dependent/outcome variable**: what we want to predict (weight)

Predicting using bivariate relationship

- Goal: what's our best guess about Y_i if we know what X_i is?
  - What's our best guess about one's weight this morning? Would it be helpful if we know how many steps she/he took yesterday?
- Terminology:
  - **Dependent/outcome variable**: what we want to predict (weight)
  - **Independent/explanatory variable**: what we are using to predict (steps)

Predicting using bivariate relationship

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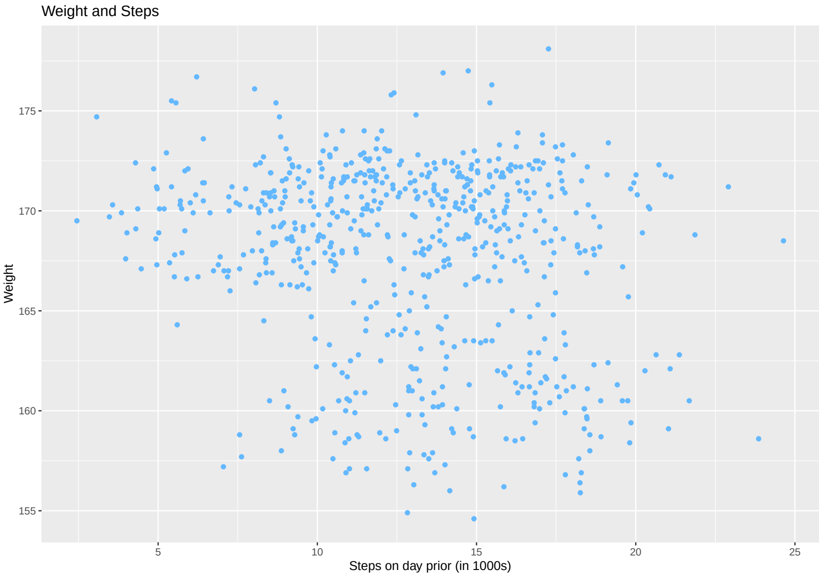
  - What's our best guess about one's weight this morning? Would it be helpful if we know how many steps she/he took yesterday?
- Terminology:
  - **Dependent/outcome variable**: what we want to predict (weight)
  - **Independent/explanatory variable**: what we are using to predict (steps)

- Load the data:

```
library(TPDdata)
health <- drop_na(health)
```

- Plot the data:

```
ggplot(health, aes(x = steps_lag, y = weight)) +
  geom_point(color = "steelblue1") +
  labs(
    x = "Steps on day prior (in 1000s)"
    y = "Weight",
    title = "Weight and Steps"
  )
```


Weight and Steps
Weight
175
170
165
160
155
5
10
15
20
25
Steps on day prior (in 1000s)

- Prediction with access to just Y: average of the Y values

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  - Need a function $y = f(x)$ that maps values of X into predictions
  - **Machine learning**: fancy ways to determine $f(x)$
- Example: what if did 5,000 steps today? What's my best guess about weight?

Start with looking at a narrow strip of X

Let's find all values that round to 5,000 steps:

```
health |>
  filter(round(steps_lag) == 5)
```

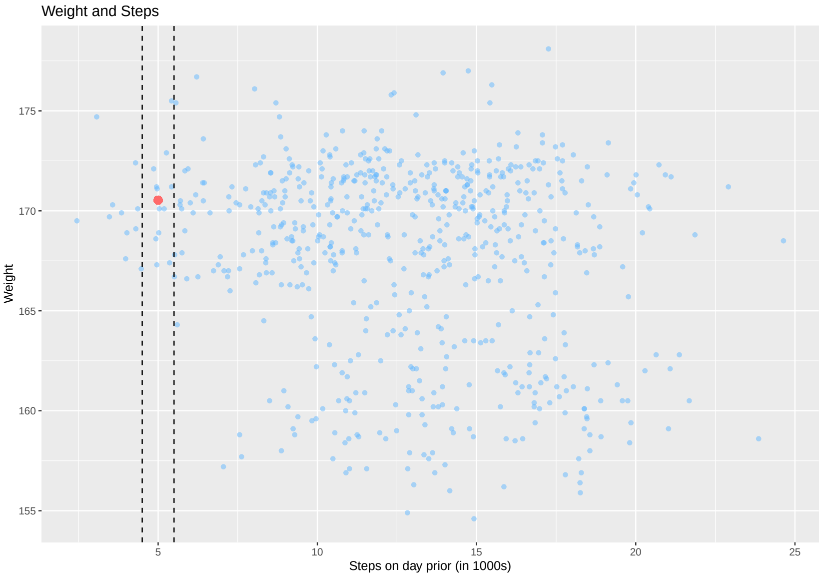
```
## # A tibble: 12 x 6
##    date       active_calories steps weight steps_lag calorie_lag
##    <date>               <dbl> <dbl>  <dbl>     <dbl>       <dbl>
##  1 2015-09-08           1111. 15.2    169.      5.02        410.
##  2 2015-12-12            728. 14.7    167.      5.36        259.
##  3 2015-12-28            430.  8.94   170.      5.19        314
##  4 2016-01-29            475.  8.26   171.      4.95        314.
##  5 2016-02-14            264.  5.42   172.      4.86        297.
##  6 2016-02-15            892. 13.1    171.      5.42        264.
##  7 2016-05-02            627. 11.8    170.      5.04        283.
##  8 2016-06-27            352.  7.21   169.      4.93        212.
##  9 2016-07-22            766. 14.8    167.      4.96        251.
## 10 2016-11-25            452   9.4    173.      5.26        295
## 11 2016-11-28            577. 11.8    171.      4.97        304.
## 12 2016-12-30            621. 12.4    176.      5.42        371.
```

Best prediction about weight for a step count of roughly 5,000 is the average weight for observations around that value:

```
mean_wt_5k_steps <- health |>
  filter(round(steps_lag) == 5) |>
  summarize(mean(weight)) |>
  pull()
mean_wt_5k_steps
```

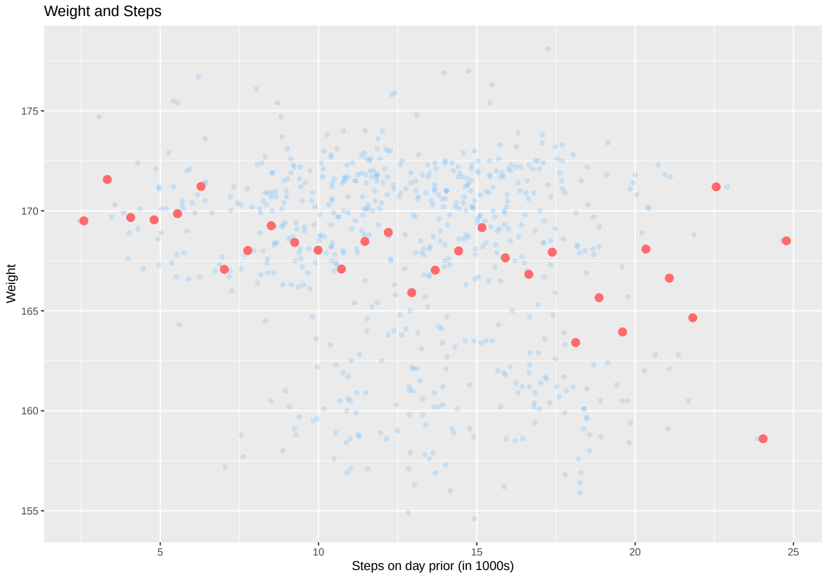
```
## [1] 170.5333
```

```r
ggplot(health, aes(x = steps_lag, y = weight)) +
  geom_point(color = "steelblue1", alpha = 0.5) +
  labs(
    x = "Steps on day prior (in 1000s)",
    y = "Weight",
    title = "Weight and Steps") +
  geom_vline(xintercept = c(4.5, 5.5), linetype = "dashed") +
  geom_point(aes(x = 5, y = mean_wt_5k_steps), color = "indianred1",
size = 3)
```


Weight and Steps
Weight
175
170
165
160
155
5
10
15
20
25
Steps on day prior (in 1000s)

We can use a `stat_summary_bin()` to add these binned means all over the scatter plot:

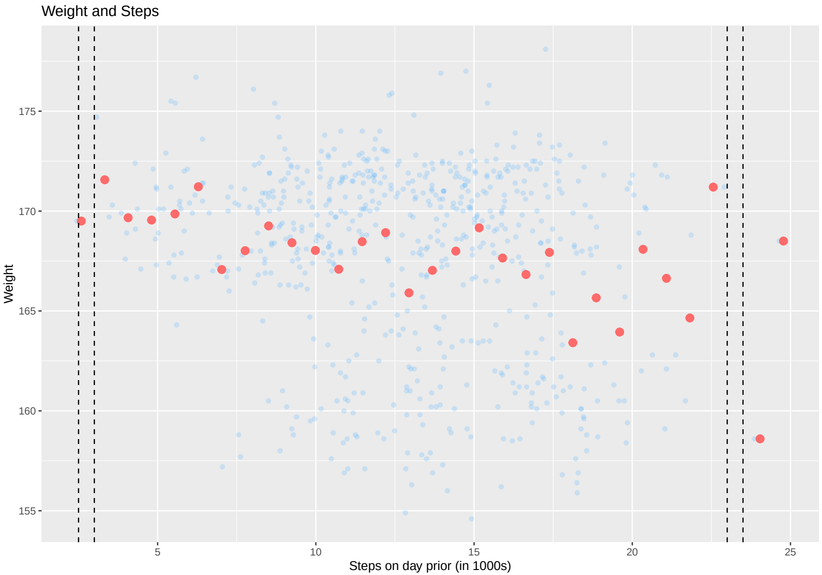
```
ggplot(health, aes(x = steps_lag, y = weight)) +
  geom_point(color = "steelblue1", alpha = 0.25) +
  labs(
    x = "Steps on day prior (in 1000s)",
    y= "Weight",
    title = "Weight and Steps"
  ) +
  stat_summary_bin(fun = "mean", color = "indianred1", size = 3,
                   geom = "point", bandwidth = 1)
```


Weight and Steps
Weight
175
170
165
160
155
5
10
15
20
25
Steps on day prior (in 1000s)

But what happens when we make the bins too small?

```
ggplot(health, aes(x = steps_lag, y = weight)) +
  geom_point(color = "steelblue1", alpha = 0.25) +
  labs(
    x = "Steps on day prior (in 1000s)",
    y= "Weight",
    title = "Weight and Steps"
  ) +
  stat_summary_bin(fun = "mean", color = "indianred1", size = 3,
                   geom = "point", bandwidth = 0.5) +
  geom_vline(xintercept = c(2.5, 3, 23, 23.5), linetype = "dashed")
```

Gaps and bumps:



Modeling with a line

- Can we smooth out these binned means and close gaps? **A model**
- Simplest possible way to relate two variables: a line

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Using a line to predict

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- Simplest possible way to relate two variables: a line

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  - Some points will be above the line, some below
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- Useful fiction: this model represents the **data generating process**
  - George Box (British statistician): ***"all models are wrong, some are useful"***

Interpreting the regression line

$$Y_i = \alpha + \beta \cdot X_i + \epsilon_i$$

- **Intercept** α: average value of Y when X is 0

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- **Intercept** α: average value of Y when X is 0
  - Average weight when I take 0 steps the day prior
- **Slope** β: average change in Y when X increases by one unit

Interpreting the regression line

$$Y_i = \alpha + \beta \cdot X_i + \epsilon_i$$

- **Intercept** α: average value of Y when X is 0
  - Average weight when I take 0 steps the day prior
- **Slope** β: average change in Y when X increases by one unit
  - Average decrease in weight for each additional 1,000 steps

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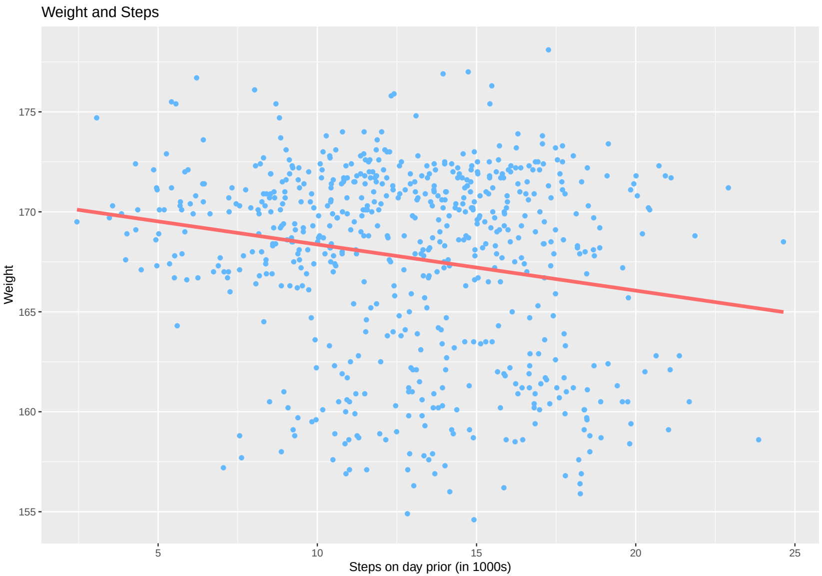
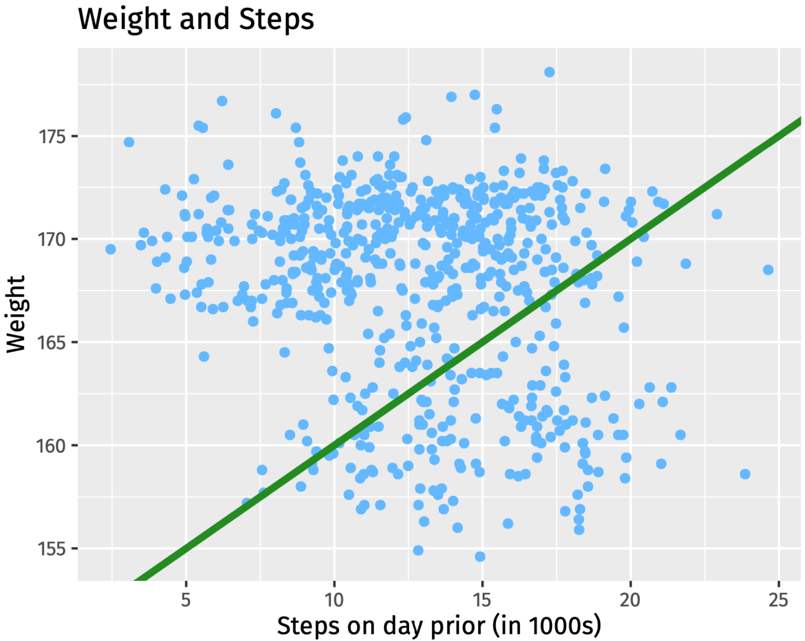
  - An **estimate** is our best guess about some parameter
- **Regression line**: $\hat{Y} = \hat{\alpha} + \hat{\beta} \cdot x$
  - Average value of Y when X is equal to x
  - Represents the best guess of **predicted value** of the outcome at x

```
ggplot(health, aes(x = steps_lag, y = weight)) +
  geom_point(color = "steelblue1") +
  labs(
    x = "Steps on day prior (in 1000s)",
    y = "Weight",
    title = "Weight and Steps")+
  geom_smooth(method = "lm", se = FALSE, color = "indianred1", size = 1.5)
```


Weight and Steps
Weight
175
170
165
160
155
5
10
15
20
25
Steps on day prior (in 1000s)


Weight and Steps
Weight
175
170
165
160
155
5
10
15
20
25
Steps on day prior (in 1000s)

Let's understand the **prediction error** for a line with intercept a and slope b

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Fitted/predicted value for unit i:

$$a + b \cdot X_i$$

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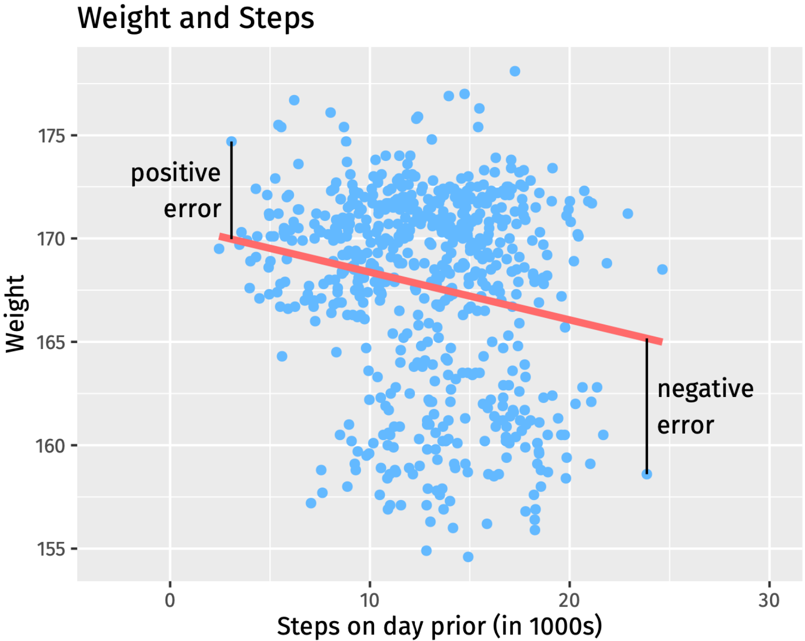
Prediction error (residual):

$$\textit{error} = \textit{actual} - \textit{predicted} = Y_i - (a + b \cdot X_i)$$


Weight and Steps
positive
error
negative
error
175
170
165
160
155
Weight
0
10
20
30
Steps on day prior (in 1000s)

- Get these estimates by the **least squares methods**

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- Minimize the **sum of the squared residuals** (SSR):

$$SSR = \sum_{i=1}^{n} (\text{prediction error}_i)^2 = \sum_{i=1}^{n} (Y_i - a - b \cdot X_i)^2$$

- Get these estimates by the **least squares methods**
- Minimize the **sum of the squared residuals** (SSR):

$$SSR = \sum_{i=1}^{n} (\text{prediction error}_i)^2 = \sum_{i=1}^{n} (Y_i - a - b \cdot X_i)^2$$

- Finds the line that minimizes the magnitude of the prediction errors!

Linear regression in R

- R will calculate least squares line for a data set using lm()

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    - Syntax: lm(y x, data = mydata)

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  - x is the name of the independent variable
  - mydata is the data.frame where they live

```r
fit <- lm(weight ~ steps_lag, data=health)
fit
```

```
##
## Call:
## lm(formula = weight ~ steps_lag, data = health)
##
## Coefficients:
## (Intercept)    steps_lag
##    170.6750      -0.2308
```

use `coef()` to extract estimated coefficients:

```
coef(fit)
```

```
## (Intercept)   steps_lag
## 170.6749706  -0.2307681
```

Interpretation: a 1-unit increase in X (1,000 steps) is associated with a decrease in the average weight of 0.231 pounds use `coef()` to extract estimated coefficients:

```
coef(fit)
```

```
## (Intercept)   steps_lag
## 170.6749706  -0.2307681
```

Interpretation: a 1-unit increase in X (1,000 steps) is associated with a decrease in the average weight of 0.231 pounds

Question: what would this model predict about the change in average weight for a 10,000 step increase in steps?

The `broom` package can provide nice summaries of the regression output

`augment()` can show fitted values, residuals and other unit-level statistics:

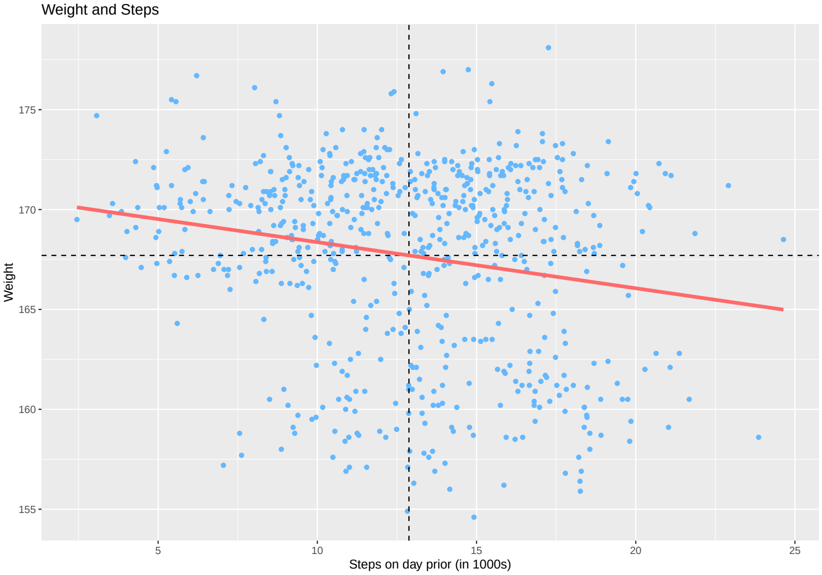
```
library(broom)
augment(fit) |>
  head()
```

```
## # A tibble: 6 x 8
##   weight steps_lag .fitted  .resid     .hat .sigma      .cooksd .std.resid
##    <dbl>     <dbl>   <dbl>   <dbl>    <dbl>  <dbl>        <dbl>      <dbl>
## 1   169.      17.5    167.  2.46    0.00369   4.68 0.000513          0.526
## 2   168       18.4    166.  1.57    0.00463   4.68 0.000264          0.337
## 3   167.      19.6    166.  1.05    0.00609   4.68 0.000154          0.224
## 4   168.      10.4    168. -0.0750 0.00217   4.68 0.000000280   -0.0160
## 5   168.      18.7    166.  1.44    0.00496   4.68 0.000238          0.309
## 6   166.       9.14   169. -2.27    0.00296   4.68 0.000349         -0.485
```

Least squares line always goes through $(\overline{X}, \overline{Y})$

```
ggplot(health, aes(x = steps_lag, y = weight)) +
  geom_point(color = "steelblue1") +
  labs(
    x = "Steps on day prior (in 1000s)",
    y = "Weight",
    title = "Weight and Steps") +
  geom_hline(yintercept = mean(health$weight), linetype = "dashed") +
  geom_vline(xintercept = mean(health$steps_lag), linetype = "dashed") +
  geom_smooth(method = "lm", se = FALSE, color = "indianred1", size = 1.5)
```

Least squares line always goes through $(\overline{X}, \overline{Y})$



Estimated slope is related to correlation:

$$\hat{\beta} = (\text{correlation of X and Y}) \times \frac{\text{SD of Y}}{\text{SD of X}}$$

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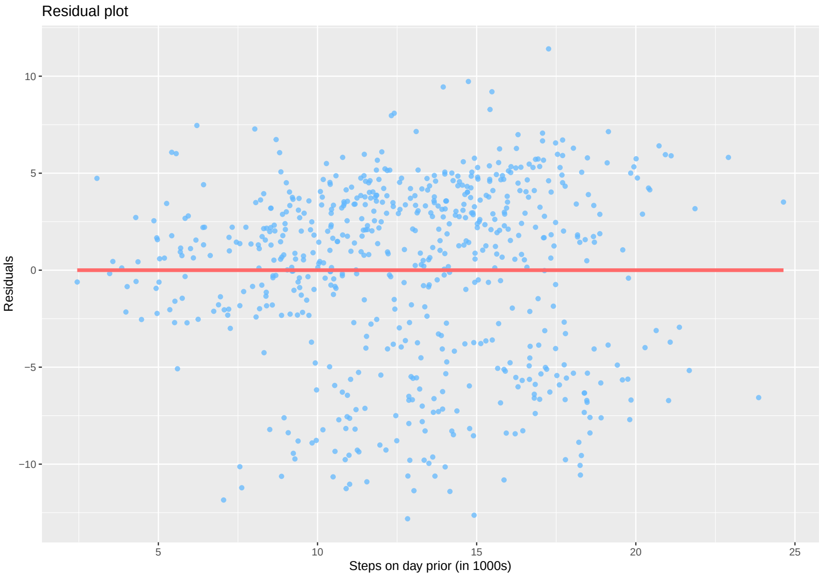
$$\hat{\beta} = (\text{correlation of X and Y}) \times \frac{\text{SD of Y}}{\text{SD of X}}$$

Mean of residuals is always 0

```
augment(fit) |>
  summarize(mean(.resid))
```

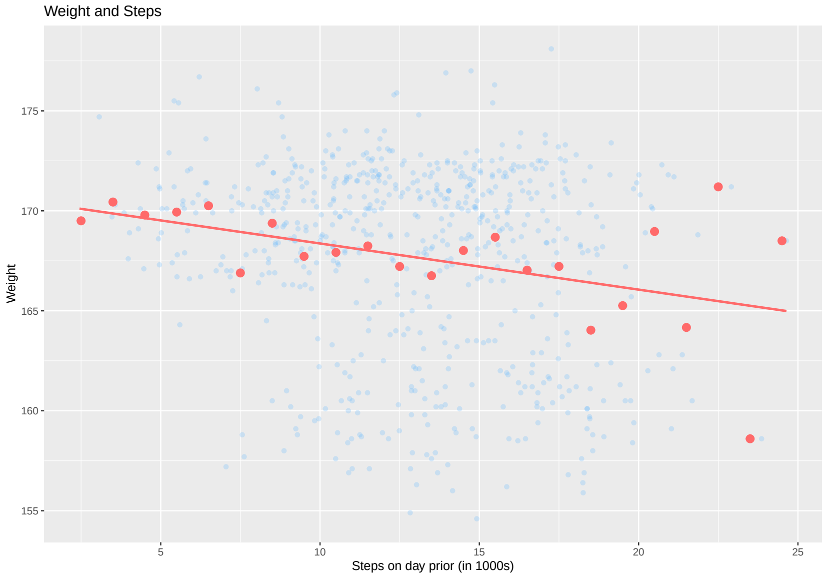
```
## # A tibble: 1 x 1
##   `mean(.resid)`
##            <dbl>
## 1      -1.21e-13
```

```r
augment(fit) |>
  ggplot(aes(x = steps_lag, y = .resid)) +
  geom_point(color = "steelblue1", alpha = 0.75) +
  labs(
    x = "Steps on day prior (in 1000s)",
    y = "Residuals",
    title = "Residual plot") +
  geom_smooth(method = "lm", se = FALSE, color = "indianred1", size = 1.5)
```


Residual plot
Residuals
Steps on day prior (in 1000s)
10
5
0
−5
−10
5
10
15
20
25

Another way to think of the regression line is a smoothed version of the binned means plot:

```
ggplot(health, aes(x = steps_lag, y = weight)) +
  geom_point(color = "steelblue1", alpha = 0.25) +
  labs(
    x = "Steps on day prior (in 1000s)",
    y = "Weight",
    title = "Weight and Steps") +
  stat_summary_bin(fun = "mean", color = "indianred1", size = 3,
                   geom = "point", binwidth = 1) +
  geom_smooth(method = "lm", se = FALSE, color = "indianred1", size = 1)
```


Weight and Steps
Weight
175
170
165
160
155
5
10
15
20
25
Steps on day prior (in 1000s)